The Volatility Of Greek Interbank Rates: A Continuous Time Analysis

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Abstract

In this paper we investigate the relationship between the volatility of Greek interbank rates and the level of rates by estimating the important CKLS interest rate model using the estimation method of (Nowman, 1997). We also estimate the interest rate models of Merton, Vasicek, CIRSR, Dothan, GBM, Brennan and Schwartz, CIRVR, and CEV models. We find the volatility of short-term rates is highly sensitive to the level of rates in Greece and is much higher than is usually assumed by these commonly used models in the financial markets.

1. Introduction

Over the last few years there has been an extensive investigation of the empirical performance of different interest rate models applied to different markets given the growing importance of interest rate models in the valuation of derivative securites. In (Chan, Karolyi, Longstaff and Sanders, 1992, thereafter CKLS) they compared a range of different single-factor term structure models for the short-term interest rate. Their major conclusion is that term structure models with volatilities more highly sensitive to the level of interest rates have a better fit to the data. Extensive comparisons have now been undertaken by for example, (Tse, 1995), (Dahlquist, 1996) and (Nowman, 1997, 1998). To date little work has been done using data from Greece. In this paper we estimate the CKLS model and the special cases by (Merton, 1973), (Vasicek, 1977), (Cox,

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Ingersoll and Ross, 1985, hereafter CIRSR), (Dothan, 1978), Geometric Brownian Motion (GBM) process of (Black and Scholes, 1973), (Brennan and Schwartz, 1980, hereafter BS), (Cox, Ingersoll and Ross, 1980, hereafter CIRVR), and the Constant Elasticity of Variance (CEV) process used in (Cox, 1975) and (Cox and Ross, 1976) using the approach of (Nowman, 1997). We find the relationship of the volatility of rates to the level of rates is much higher than is assumed by these currently used models in the financial markets.

Section 1 specifies the interest rate models estimated and method of estimation. Section 2 describes the data and Section 3 presents the empirical results. Section 4 contains a summary and concluding remarks.

2. Interest Rate Models and Estimation.

The general stochastic differential equation used by CKLS of the interest rate is represented by equation (1) below. Historically, it has been observed that periods of high interest rates have led to periods of low interest rates and vice versa. This process is known as mean reversion. Furthermore, it has been observed that a random element exists in the movements of interest rates which is generally dependent on the current level of interest rates. Both these important historical characteristic are incorporated by the CKLS model below by allowing the conditional mean and variance to depend on the level r.

$$dr(t) = \{\alpha + \beta r(t)\}dt + \sigma r^{\gamma}(t)dZ \qquad (t \geq 0) \qquad (1)$$

where $\{r(t),\ t > 0\}$ is a real continuous time random process, α, β, γ, and σ are unknown structural parameters and dZ is a Wiener process. From the CKLS model we can obtain the important special cases: Merton$(\beta = 0, \gamma = 0)$, Vasicek$(\gamma = 0)$, CIR$(\gamma = 0.5)$, Dothan$(\alpha = 0, \beta = 0, \gamma = 1)$, GBM$(\alpha = 0, \gamma = 1)$, BS$(\gamma = 1)$, CIRVR$(\alpha = 0, \beta = 0, \gamma = 3/2)$, and CEV$(\alpha = 0)$. (See CKLS for a more detailed analysis of the use of these models).

To estimate the CKLS model historically we use the approach of (Nowman, 1997). The discrete model used by (Nowman, 1997) is given by (2) below

$$r(t) = e^{\beta}r(t-1) + \frac{\alpha}{\beta}\left(e^{\beta} - 1\right) + \eta_t \qquad (t = 1,2,\ldots,T) \qquad (2)$$

where η_t $(t = 1,2,\ldots,T)$ satisfied the conditions given in (Nowman, 1997). Following (Nowman, 1997) we let $\mathrm{L}(\theta)$ be minus twice the logarithm of the Gaussian likelihood function where the complete vector of parameters is $\theta = [\alpha, \beta, \gamma, \sigma^2]$. Then the Gaussian estimates (see Bergstrom, 1990) are obtained from equation (3) below where m_{tt}^2 was given in (Nowman, 1997).

$$L(\theta) = \sum_{t=1}^{T} \left[2 \log m_{tt} + \frac{\left\{ r(t) - e^{\beta} r(t-1) - \frac{\alpha}{\beta}\left(e^{\beta} - 1\right)\right\}^2}{m_{tt}^2} \right] \qquad (3)$$

3. Data

Table I

Summary Statistics: Greek Interbank Data
July 1994 - June 1998

Variables	*T*	*Mean*	*Standard Deviation*	ρ_1,	ρ_2,	ρ_3,	ρ_4,	ρ_5,	ρ_6,
1-Month									
$r(t)$	1039	0.0148	0.0370	0.92	0.85	0.82	0.80	0.75	0.68
$dr(t)$	1038	-0.0001	0.0143	-0.09	-0.20	-0.08	0.20	0.10	-0.05
2-Month									
$r(t)$	1039	0.1487	0.0357	0.92	0.86	0.84	0.83	0.79	0.74
$dr(t)$	1038	-0.0001	0.0134	-0.11	-0.26	-0.08	0.18	0.12	-0.05
3-Month									
$r(t)$	1039	0.1489	0.0350	0.93	0.87	0.85	0.85	0.83	0.80
$dr(t)$	1038	-0.0001	0.0120	-0.07	-0.30	-0.11	0.11	0.11	0.01
6-Month									
$r(t)$	1039	0.1491	0.0348	0.96	0.94	0.92	0.92	0.90	0.88
$dr(t)$	1038	-0.0001	0.0834	-0.21	-0.13	-0.11	0.10	0.05	0.01

The short-term interest rates used in this study are daily one, two, three and six month Greek interbank rates obtained from *Datastream*. Table I reports the descriptive statistics for the Greek data. The table displays the means, standard deviations, and first six autocorrelations of the rates and daily changes in the rates The mean and standard deviations of the different series are as follows: one month mean is 14.823% with a standard deviation of 3.708%; two month mean is 14.870% with a standard deviation of 3.579%; three month mean is 14.898% with a standard deviation of 3.509% and lastly the six month mean is 14.916% with a standard deviation of 3.483%. Although the autocorrelations in the interest rate level for the different maturities decay slowly, those of the day-to-day changes are generally small.

4. Results

Table II

Estimates of Short Term Interest Rate Models: 1-Month

Model	α	β	σ^2	γ	*Log Likelihood*	χ^2 *Test*	*df*
CKLS	0.0025	-0.0153	0.3395	2.1245	4314.8007		
	(0.0015)	(0.0116)	(0.1487)	(0.1125)			
Merton	-0.0001	0.0	0.0002	0.0	3888.0055	853.5904	2
	(0.0004)		(<0.0001)				
Vasicek	0.0121	-0.0823	0.0002	0.0	3910.2245	809.1524	1
	(0.0019)	(0.0129)	(<0.0001)				
CIRSR	0.0070	-0.0482	0.0009	0.5	4121.9156	385.7702	1
	(0.0017)	(0.0121)	(<0.0001)				
Dothan	0.0	0.0	0.0051	1.0	4234.8081	159.9852	3
			(0.0002)				
GBM	0.0	0.0013	0.0050	1.0	4234.9743	159.6528	2
		(0.0021)	(0.0002)				
BS	0.0044	-0.0301	0.0052	1.0	4238.6212	152.3590	1
	(0.0016)	(0.0116)	(0.0002)				
CIRVR	0.0	0.0	0.0313	1.5	4291.9895	45.6224	3
			(0.0013)				
CEV	0.0	0.0033	0.3461	2.1337	4313.5487	2.5040	1
		(0.0019)	(0.1454)	(0.1081)			

In Table II we present the Gaussian coefficient estimates (standard errors in parentheses) and maximised log likelihoods for the unrestricted and nested models. The models explanatory power following (Nowman, 1997) compared to the unrestricted model is compared using the maximised Gaussian log likelihood function values. For the one month maturity based on maximised Gaussian log likelihood values compared to the unrestricted model, the CEV model performs the best followed by the CIRVR, BS, GBM, Dothan, CIRSR, Vasicek and Merton models. The results imply an unrestricted estimate of $\gamma = 2.1245$ for the one month rate. This implies the volatility of rates is highly dependent on the level of rates and is far higher than assumed by these well known models. The unrestricted model implies weak evidence of mean reversion in the one month rate. The important Vasicek, CIRSR, and BS models display evidence of mean reversion also. Based on the χ^2 likelihood ratio test under the null hypothesis that the nested models restrictions are vaild the results imply that we can reject all models except the CEV model.

Table III

Estimates of Short Term Interest Rate Models: 2-Month

Model	α	β	σ^2	γ	*Log Likelihood*	χ^2 *Test*	*df*
CKLS	0.0022	-0.0135	0.3029	2.1067	4330.6091		
	(0.0015)	(0.0122)	(0.1281)	(0.1088)			
Merton	-0.0001	0.0	0.0001	0.0	3957.1561	746.9060	2
	(0.0004)		(<0.0001)				
Vasicek	0.0114	-0.0777	0.0001	0.0	3978.3050	704.6082	1
	(0.0018)	(0.0121)	(<0.0001)				
CIRSR	0.0066	-0.0454	0.0008	0.5	4278.5228	104.1726	1
	(0.0016)	(0.0113)	(<0.0001)				
Dothan	0.0	0.0	0.0048	1.0	4257.6245	145.9692	3
			(0.0002)				
GBM	0.0	0.0012	0.0048	1.0	4257.7804	145.6574	2
		(0.0021)	(0.0002)				
BS	0.0042	-0.0281	0.0049	1.0	4261.3338	138.5506	1
	(0.0016)	(0.0115)	(0.0002)				
CIRVR	0.0	0.0	0.0299	1.5	4309.8914	41.4354	3
			(0.0013)				
CEV	0.0	0.0032	0.3096	2.1161	4329.5078	2.2026	1
		(0.0019)	(0.1271)	(0.1058)			

Turning to Table III for the two month maturity based on maximised Gaussian log likelihood values compared to the unrestricted model, the CEV model performs the best followed by the CIRVR, BS, GBM, Dothan, CIRSR, Vasicek and Merton models. The results imply an unrestricted estimate of $\gamma = 2.1067$ for the two month rate impliying the volatility of rates is again highly dependent on the level of rates. Based on the χ^2 likelihood ratio test under the null hypothesis that the nested models restrictions are vaild the results imply that we can reject all models except the CEV model.

Table IV

Estimates of Short Term Interest Rate Models: 3-Month

Model	α	β	σ^2	γ	*Log Likelihood*	χ^2 *Test*	*df*
CKLS	0.0020	-0.0122	0.1946	2.0014	4345.0512		
	(0.0013)	(0.0099)	(0.0850)	(0.1131)			
Merton	-0.0001	0.0	0.0001	0.0	4069.7785	550.5454	2
	(0.0003)		(<0.0001)				
Vasicek	0.0096	-0.0656	0.0001	0.0	4087.9506	514.2012	1
	(0.0016)	(0.0107)	(<0.0001)				
CIRSR	0.0059	-0.0407	0.0007	0.5	4221.5764	246.9496	1
	(0.0015)	(0.0109)	(<0.0001)				
Dothan	0.0	0.0	0.0045	1.0	4294.0727	101.9570	3
			(0.0002)				
GBM	0.0	0.0010	0.0045	1.0	4294.1921	101.7182	2
		(0.0021)	(0.0002)				
BS	0.0038	-0.0260	0.0046	1.0	4297.5741	94.9542	1
	(0.0015)	(0.0106)	(0.0002)				
CIRVR	0.0	0.0	0.0285	1.5	4332.2455	25.6114	3
			(0.0013)				
CEV	0.0	0.0028	0.2020	2.0145	4344.1057	1.8910	1
		(0.0019)	(0.0908)	(0.1161)			

Turning to Table IV for the three month maturity based on maximised Gaussian log likelihood values compared to the unrestricted model, the CEV model performs the best followed by the CIRVR, BS, GBM, Dothan, CIRSR, Vasicek and Merton models. The results imply an unrestricted estimate of $\gamma = 2.0014$ for the three month rate. Based on the χ^2 likelihood ratio test the results imply that we can reject all models except the CEV model.

Table V

Estimates of Short Term Interest Rate Models: 6-Month

Model	α	β	σ^2	γ	*Log Likelihood*	χ^2 *Test*	*df*
CKLS	0.0032	-0.0222	0.0065	1.2334	4578.3586		
	(0.0011)	(0.0078)	(0.0022)	(0.0897)			
Merton	-0.0001	0.0	0.0001	0.0	4452.0786	252.5600	2
	(0.0003)		(<0.0001)				
Vasicek	0.0049	-0.0339	0.0001	0.0	4462.3231	232.0710	1
	(0.0011)	(0.0075)	(<0.0001)				
CIRSR	0.0041	-0.0284	0.0004	0.5	4540.5079	75.7014	1
	(0.0011)	(0.0076)	(<0.0001)				
Dothan	0.0	0.0	0.0026	1.0	4569.7246	17.2680	3
			(0.0001)				
GBM	0.0	0.0005	0.0026	1.0	4569.7713	17.1746	2
		(0.0015)	(0.0001)				
BS	0.0035	-0.0240	0.0026	1.0	4574.8676	6.9820	1
	(0.0011)	(0.0079)	(0.0001)				
CIRVR	0.0	0.0	0.0180	1.5	4570.1246	16.4680	3
			(0.0007)				
CEV	0.0	0.0011	0.0070	1.2574	4574.0312	8.6548	1
		(0.0016)	(0.0024)	(0.0890)			

Turning to Table V for the six month maturity based on maximised Gaussian log likelihood values compared to the unrestricted model, the CEV model performs the best followed by the BS, CIRVR, GBM, Dothan, CIRSR, Vasicek and Merton models. The results imply an unrestricted estimate of $\gamma = 1.2334$ for the six month rate which indicates that the volatility of rates is less dependent on the level of rates compared to the shorter maturities considered above.

Based on the χ^2 likelihood ratio test the results imply that we can reject all models.

5. Conclusions

In this paper we have investigated the relationship between the volatility of Greek interbank rates and the level of rates by estimating the important CKLS interest rate model. We also estimated the interest rate models of Merton, Vasicek, CIRSR, Dothan, GBM, BS, CIRVR, and CEV. We find the volatility of short-term rates is highly sensitive to the level of rates in Greece and is much higher than is usually assumed by these commonly used models in the financial markets.

References

1. Bergstrom, A. R, 1990, *"Continuous time econometric modelling"* (Oxford University Press, Oxford).
2. Black, F. and M. Scholes, 1973, "The pricing of options and corporate liabilities", *Journal of Political Economy* 81, 637-654.
3. Brennan, M. J. and E. S. Schwartz, 1980, Analyzing convertible bonds", *Journal of Financial and Quantitative Analysis* 15, 907-929.
4. Chan, K. C., G. A. Karolyi, F. A. Longstaff, and A. B. Sanders, 1992, "An empirical comparison of alternative models of the short-term interest rate", *Journal of Finance* 47, 1209-1227.
5. Cox, J. C, 1975, "Notes on option pricing I: constant elasticity of variance diffusions", Working paper, Stanford University.
6. Cox, J. C. and S. A.Ross, 1976, "The valuation of options for alternative stochastic processes", *Journal of Financial Economics* 3, 145-166.
7. Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1980, "An analysis of variable rate loan contracts", *Journal of Finance* 35, 389-403.
8. Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, "A theory of the term structure of interest rates", *Econometrica* 53, 385-407.
9. Dahlquist, M, 1996, "On alternative interest rate processes", *Journal of Banking and Finance* 20, 1093-1119.

10. Dothan, U. L, 1978, "On the term structure of interest rates",*Journal of Financial Economics* 6, 59-69.

11. Merton, R. C, 1973, "Theory of rational option pricing", *Bell Journal of Economics and Management Science* 4, 141-183.

12. Nowman, K. B, 1997, "Gaussian estimation of single-factor continuous time models of the term structure of interest rates", *Journal of Finance* 52, 1695-1706.

13. Nowman, K. B, 1998, "Continuous time short term interest rate models", *forthcoming in Applied Financial Economics*.

14. Tse, Y. K, 1995, "Some international evidence on the stochastic behavior of interest rates", *Journal of International Money and Finance* 14, 721-738.

15. Vasicek, O. A, 1977, "An equilibrium characterization of the term structure", *Journal of Financial Economics* 5, 177-188.