The Mathematical Model of the Law on the Correlation of Unique Competencies with the Emergence of New Consumer Markets

A.A. Chursin¹, R.V. Shamin, L.A. Fedorova

Abstract:

The article deals with the applied aspects of assessing the influence degree of unique competencies on the process of creating innovative technologies and forming new consumer markets.

It also builds the economic model, which demonstrates the presence of this correlation. Paper formulates the law on spiral dependence of the competences level on the emergence of new markets.

The authors proposed mathematical instruments based on simulation modeling, which confirms the objectivity of the derived law.

Keywords: knowledge economy, unique competences, innovative technologies, new consumer markets, model, law.

JEL Classification: K10, O10, O31

¹ Peoples' Friendship University of Russia (RUDN University)
6 Miklukho-Maklaja St, Moscow, 117198, Russian Federation
Introduction

The analysis of the common patterns that characterize the correlation of unique competences with the formation of new consumer markets showed that the process of their emergence is typical for the formation of a national knowledge economy, the main elements of which are the competences of corporations and innovative technologies (Amidon, Formica, and Mercier-Laurent, 2005; Dzhukha et al., 2017; Ivanova et al., 2017).

The formation of intellectual economics is targeted at developing new products, services and rapid growth of new markets (Powell and Snellman 2004), which in turn generates the demand for higher level of competences and introduction of innovative technologies (Rooney, Hearn and Ninan 2005). Thus, the economy witnesses a spiral trajectory of its development, which is explained by the law deduced by the authors on the relationship between the growth of competence level and the emergence of new markets. In its turn the law can not only illustrate new economic patterns, but also it can be used as a postulate of a microeconomic theory describing specific features of the knowledge-based enterprises functioning.

The model “Competencies-innovations-markets”

The modern world economy is getting more and more oriented at a new economic type - knowledge-based economy (Meusburger, Glückler and Meskioui, 2013). Today, the resources that determine the further trajectory of the national economy development are not natural wealth, productive capacities, or even labor resources, but the unique competencies of corporations and employees. They play a crucial role both at microeconomic and macroeconomic level.

The development of unique competencies produces a significant synergetic effect in the economy, which manifests itself not only in the industry but also in education, as the demand for the competencies and high level of training generates the supply of new educational services. In addition, the development of competences and derived from them new technologies lays the groundwork for new consumer markets development. Competition in the field of high technologies and unique competences has mechanisms different from traditional industries responding more quickly to competitors’ activity, which motivate companies to change rapidly and effectively their behavior.

Let us look at the model of correlation between competences development and consumer markets, the correlation between which leads to spiral trajectory sustaining innovative growth of the economy. Schematically, this can be depicted in Figure 1.
Thus, the effective functioning of knowledge-based enterprises shapes the demand for innovative products. Consequently the demand for key competences arises in new industries. Naturally, the increasing demand encourages the growth and application of key competences. All this contributes to the development of fundamental science, generates applied developments, and form the basis of innovative technologies, which are subsequently translated into the creation of new products or services.

The aim of commercializing innovative technologies makes high technological companies create new markets and promote their new products their. As a result, these new consumer markets for innovative products trigger the rapid growth of the corresponding productions, in which a large number of new key competences are formed that in their turn result in emergence of new products and ultimately new consumer markets (Chursin, and Shamin, 2011; Akopova and Przhedetskaya, 2016; Kormishkin et al., 2016).

The model “Competence-Innovations-Markets” demonstrates that the higher level of competences encourages the widening of consumer markets at the expense of new products meeting new needs. In so doing the market mechanism causes the surge in
investments into breakthrough technologies, which are used for these products manufacturing. According to the lows of innovative economics these innovations greatly extend competences in this field. Undoubtedly, this surge takes place only for some time while consumer markets are growing impressively, but according to the economic law of marginal market saturation when they get saturated at some point of time, the growth sharply slows as the technologies that used to be innovative become ordinary. In such a situation for the continued growth the economy needs the emergence of newer innovative technologies that become a prologue for further growth of consumer markets and, respectively competences enhancement (Vasilyev and Chursin, 2011).

The provided example of the correlation between new competences and new consumer markets can be described in the form of a closed economic model, which describes the internal processes of this correlation. This model can be represented in the form of the general law on the emergence of new markets on the basis of new key competences and depicts the spiral trajectory of “competences-resources-products-needs-competences”.

In this paper we formulate and provide mathematical justification of the law on the correlation between competences and new market emergence: The process of unique competencies development in high-tech enterprises forms a system of internal intellectual resources, the use of which provides an abrupt trajectory of the innovative technologies development for the production of more modern products, the implementation of which involves the creation of new consumer markets.

Thus, we suggest the hypothesis that as a result of the constantly recurring cycle of "growing markets - shrinking markets" a modern high-tech economy accumulates a sufficient stock of competencies, which in its turn forms the prerequisites for the emergence of a new cycle of markets and competencies growth.

**Interpretation of the law through the mathematical model**

The proposed model is described with some dynamic parameters that are interdependent and changeable as time goes on, namely:

- The level of technological competences;
- The level of innovative technologies;
- The level of new products;
- The level of market development.

We use in the model the dimensionless numbers that is why not particular values of a parameter are important but their dynamics. Consider the details of the parameters listed above. The level of technological competences describes the existing average level of technological competences that can be used in high-tech industries to develop innovative technologies. Another important parameter in the model is the level of innovative technologies; this parameter depends on the parameter describing
the level of competences. It describes the degree of innovative technologies introduction into the manufacturing processes. While calculating the dynamics of this parameter we should take into account the factors of the innovative technologies obsolescence when newer technologies replace the existing ones.

The parameter describing the level of new products is closely linked to the level of innovative technologies. The introduction of innovative technologies enables to not only improve the qualitative properties of the products or lower their costs but also provides the opportunity to design new products and services. Naturally, this level is a numerical indicator, which is used to specify the number of new products in the consumer markets. To calculate this parameter, you need to use econometric information, as well as comparative information about the indicators of products.

The most important parameter in our model is the one that describes the level of consumer markets. This level reflects the creation and development of new markets, emerging from the supply of new products. Accordingly, there is a positive feedback, which causes a further demand for innovative technologies, as well as the development of unique competences. For constructing the mathematical model of the law under discussion it is necessary to consider the formalizing of basic concepts. As we investigate the dynamic model we should introduce “time” into it. Typically, economic models include discrete time, since many processes develop over long time, but for qualitative results it is convenient to consider continuous time, since in this case more compact mathematical models are obtained.

\[ t \in [0,T], \ 0 < T < \infty \]

We consider the case of finite time interval. The characteristics of the mathematical model for describing the law on the correlation between competences and new markets are the following variables:

C(t) - a quantitative indicator of financing competencies;
T(t) - a quantitative indicator of innovative technologies;
P(t) - the quantitative indicator of products;
M(t) - a quantitative indicator of new markets.

We consider the following system of differential equations:

\[
\begin{align*}
\dot{C}(t) &= F_1(C(t),T(t),P(t),M(t)) \\
\dot{T}(t) &= F_2(C(t),T(t),P(t),M(t)) \\
\dot{P}(t) &= F_3(C(t),T(t),P(t),M(t)) \\
\dot{M}(t) &= F_4(C(t),T(t),P(t),M(t))
\end{align*}
\]
The apparatus of differential equations used for formalizing the law on the competences mutual influence on consumer markets makes it possible to construct a qualitative mathematical model for investigating the dynamics of these quantities in different contexts.

When constructing formal bases for the model, we need to choose the basic numerical characteristics to describe the corresponding economic quantities, so we propose to consider dimensionless quantities that qualitatively describe the state of economic variables. Consequently, the numerical values of the functions \( C(t) \), \( T(t) \), \( P(t) \) and \( M(t) \) represent the values of integral indicators for describing the level of competence, the state of technology, products and consumer markets.

To obtain the qualitative description of mutual dynamics of competences level and the growth of consumer markets, it is more convenient to consider differential equations. The case of continuous time can be replaced with a discrete one, then in this case we can rewrite the differential equations used in the form of difference equations. The dynamics of indicators before us to a first approximation can be described by means of a linear model:

\[
\begin{align*}
\dot{C}(t) &= A_C C(t) + A_{MC} M(t) \\
\dot{T}(t) &= A_T T(t) + A_{CT} C(t) \\
\dot{P}(t) &= A_P P(t) + A_{TP} T(t) \\
\dot{M}(t) &= A_M M(t) + A_{PM} P(t)
\end{align*}
\]

In this model we consider a system of linear differential equations in which each dynamic variable has a certain diffusion coefficient that reflects an objective decline in every indicators over time, since without appropriate management indicators decrease due to the actions of competitors and the overall development of the economy and scientific and technical progress.

While modeling it is necessary to stress the fact that each subsequent indicator in it changes under the influence of the previous one, and the last indicator, the indicator of new markets, influences the initial indicator of the competences level ensuring the cyclical economic development. The main conditions of model formation are the following:

- \( A_C < 0, \ A_T < 0, \ A_P < 0, \ A_M < 0 \)

Which means that for every key indicator the model has time diffusion leading to a constant decline in the values of these indicators.

- \( A_{MC} > 0, \ A_{CT} > 0, \ A_{TP} > 0, \ A_{PM} > 0 \)

Which means that every key indicator is characterized by a positive dependence, which ensures their spiraling rise that reflects the main component of the law under
consideration. It is to be noted that the system of differential equations above do not take into consideration external impacts on the dynamics of indicators.

From this point on it is necessary to consider the following heterogeneous system of differential equations, which takes into account the external impact on the dynamics of integrated indicators:

\[
\begin{align*}
\dot{C}(t) &= A_C C(t) + A_{MC} M(t) + F_C(t) \\
\dot{T}(t) &= A_T T(t) + A_{CT} C(t) + F_T(t) \\
\dot{P}(t) &= A_P P(t) + A_{TP} T(t) + F_P(t) \\
\dot{M}(t) &= A_M M(t) + A_{PM} P(t) + F_M(t)
\end{align*}
\]

Here, the functions on the right-hand side express an external effect on the dynamics of integral indicators. These functions can be deterministic if we consider a situation of purposeful management, and also they can contain stochastic components in case of considering random external influences on dynamics of the considered indicators. The proposed linear dynamic system for describing the mutual influence of the competences level on the consumer markets development can only describe the period of increase in these values, therefore, to reflect cyclical phenomena when developing the level of competences and consumer markets we need to consider nonlinear dynamic models.

Nonlinear effects in the development of consumer markets are linked with the fundamental laws on every product marginal utility. The phenomenon of consumer markets saturation restricts the corresponding growth in competence indicators, as saturation of consumer markets reduces the operating profit of enterprises and limits additional investments in the production development and in the increasing of competencies level. In addition, the innovative technologies have their own saturation, which arises when the innovation capabilities of technologies are exhausted.

The analysis of literature (Gurevich, Shamin, and Tikhomirov, 2013) revealed that cyclic phenomena have two distinct stages - growth and decline and can be described with the help of a nonlinear effect of hysteresis. In nonlinear dynamic systems the effect of hysteresis leads to the situation when the coefficients of equations change discontinuously if the values (solutions to the equation) reach critical levels. Let us describe the hysteresis function, which we use in our dynamic model to describe the cycles of development of consumer markets and the level of key competences. For the dynamic variable \( Y(t) \), we introduce the following hysteresis function:
We will consider four hysteresis functions $H_C$, $H_T$, $H_P$, $H_M$ for the equation for competences, technologies, products and consumer market. Thus, to describe the law of correlation between competences level and consumer market we can use the following dynamic model:

$$
H[Y(t)]=\begin{cases}
H_{\text{min}}, & Y(t)>T_{\text{max}} \\
H_{\text{max}}, & Y(t)<T_{\text{min}}
\end{cases}
$$

As a result we obtain the model that is able to describe the cyclical dynamic of quantities “competences-technologies-products-markets”.

**Application of the model of correlation between competence level and the expansion of consumer markets**

To demonstrate the application of proposed mathematical model we use the simulation modeling based on a system of functional differential equations. In this system we use random factors, which we randomly simulate in the course of numerical calculation, namely we use the parameters presented in Table 1

*Table 1. Characteristics of model parameters*

<table>
<thead>
<tr>
<th>Parameter identifier</th>
<th>Parameter value</th>
<th>Economic value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_C</td>
<td>1</td>
<td>Diffusion coefficient of competence level</td>
</tr>
<tr>
<td>A_T</td>
<td>1</td>
<td>Diffusion coefficient of technologies</td>
</tr>
<tr>
<td>A_P</td>
<td>1</td>
<td>Diffusion coefficient of products</td>
</tr>
<tr>
<td>A_M</td>
<td>1</td>
<td>Diffusion coefficient of markets</td>
</tr>
<tr>
<td>H_min</td>
<td>0.1</td>
<td>Low value of hysteresis</td>
</tr>
<tr>
<td>H_max</td>
<td>1</td>
<td>Lower value of hysteresis</td>
</tr>
<tr>
<td>T_min</td>
<td>1</td>
<td>Upper value of hysteresis</td>
</tr>
<tr>
<td>T_max</td>
<td>10</td>
<td>Upper bound of hysteresis</td>
</tr>
<tr>
<td>Fx</td>
<td>~R(0.7,1.7)</td>
<td>Random influence</td>
</tr>
<tr>
<td>C(0)</td>
<td>1</td>
<td>Initial value of competence level</td>
</tr>
<tr>
<td>T(0)</td>
<td>1</td>
<td>Initial value of technologies index</td>
</tr>
<tr>
<td>P(0)</td>
<td>1</td>
<td>Initial value of products index</td>
</tr>
<tr>
<td>M(0)</td>
<td>1</td>
<td>Initial value of markets index</td>
</tr>
</tbody>
</table>
Using these parameters, we can numerically solve the system of functional-differential equations for the purpose of simulation. We use the numerical method to solve Runge-Kutta differential equations of fourth order. As the result of this modeling we obtain the following Figures 2-5.

Figures 2-5 illustrate the solution to differential equations, reflecting irregular periodical fluctuations of the indicators, which are caused by the impact of random factors. In Fig. 6 one graph simultaneously shows the results of simulation modeling for the four selected indices, on the basis of this we can speak about a certain synchronization of periods of growth and decline in these quantities.
Figure 6. Consolidated graph of simulation modeling of the law indices

By this means, it can be stated that the demonstrated cyclicity reflects the correlation between the competences level and the growth of the consumer market throughout the time period under consideration. It is noteworthy that the proposed model applies the constant values in hysteresis functions, therefore at different stages of the cycles; approximately the same maximum values of the indices were obtained. In fact, the maximum values of hysteresis functions tend to increase in course of time. Moreover, the economic growth is provided by the sophisticated technologies used to design new products. Respectively, hi-tech companies need to constantly increase their investments into competences. Consider the results of simulation modeling in the case when there is a constant increase in the maximum values of the hysteresis functions, using a linear trajectory:

\[ H_{\text{min}}(t) = \alpha t + H_{\text{min}}(0) \]

and

\[ H_{\text{max}}(t) = \beta t + H_{\text{max}}(0) \]

As the result of this simulation modeling, we obtain Figures 7-8.
The results in Figures 7-8 show that under the conditions of hysteresis functions growth, the influence of random factors decreases, at the same time they demonstrate the cyclical nature of changes in indices.

Simulation modeling shows that the proposed model for describing the correlation between the increase in competency level and the growth of consumer markets reflects various modifications in the dynamics of the basic values in the law "competences-technologies-products-markets". We can witness a certain cyclicity of these values changes, which describes the spiral growth of high-tech products markets based on innovative technologies that arise with increasing competences (Chursin, Shamin, and Kokuytseva 2012; Chursin and Makarov 2015; Chursin, Tyulin and Yudin 2016; Chursin and Vlasov 2016; Anikina, Gukova, Golodova and Chekalkina, 2016; Bashmakov, Popov, Zhedyaevskii, Chikichev and Voyakin, 2015; Frank, Mashevskaya, and Ermolina, 2016; Ovchinnikov, Kozenko, Bichkov, Kabanov, and Karpova, 2015)

Law on cyclical correlation of competences level with the market growth

Consider balance equations for deriving a mathematical description of the law of the cyclical correlation between the level of competences and the growth of consumer markets. The first balance equation can be written in the following form:

\[ M(t) = K(t)IC(t-h), \]

IC (t) is the level of financing the increase in key technological competences for the creation of innovative technologies at time t. We consider both continuous time and discrete change of time depending on the problem posed.

M (t) is a composite indicator of the consumer market, which is created as a result of the release of new products.
K (t) is a transition coefficient, the economic meaning of which is that it shows the growth of the consumer market, depending on the increase in financing the creation of key technological competences.

h > 0 is a time lag that reflects the fact that there is a time lag in the impact of financing key competences on the growth of the consumer market.

The second balance equation is written as follows:

\[ IC(t) = L(t) M(t) \]

where

L (t) is the coefficient of increasing financing the development of unique technological competences, depending on the increase in the consumer market indicator.

These balance equations describe a linear situation, although in reality these correlation should be written in a nonlinear form, since K (t) and L (t) depend not only on time, but also on the values of IC and M.

Comparing these balance equations, we can derive a common balance equation:

\[ \frac{1}{L(t)} IC(t) = K(t) IC(t - h) \]

It is more convenient to write this equation without the division operation in the following form:

\[ IC(t) = L(t) K(t) IC(t - h) \]

Consider the economic meaning of this equation and variants of its application in a practical assessment of the growth of financing the development of key technological competences. First, we have to note that this equation establishes the function of financing in innovative technologies and competences, depending on the initial conditions - the initial financing. Naturally, in a real situation this financing can depend on managerial decisions, but then the violation of balance equations means the inefficiency of investments.

Further one the most important indices in the balance equation is the condition of:

\[ L(t) K(t) > 1, \quad t \geq 0 \]

The fulfillment of this condition means continuous growth of financing the development of key technological competences. On the one hand the violation of
this condition shows that there is a constant fall in financing the development of key competences and, accordingly, the fall in consumer market indices.

As there is a cycle of growth and decline in consumer markets, the transition coefficients \( L(t) \) and \( K(t) \) must change their values. A typical situation corresponds to the following conditions on the coefficients:

\[
L(t) < 1 \\
K(t) > \frac{1}{L(t)}
\]

This situation corresponds to the fact that financing the development of key competences is declining, but the growth of consumer markets is quite large. This situation is typical, because even with the constant growth of consumer markets, the financing of innovative technologies can reduce. Taking into account the effects of the delay in the growth of consumer markets from the amount of financing the development of key competences and technologies, this situation has an economic justification.

The linear models considered are adequate only at certain stages of growth (or decline) in consumer markets, since linear models cannot reflect the situation of a sharp change in the economic situation, which can vary greatly in the markets of high-tech products. Therefore, it is necessary to consider more complex models that take into account the nonlinearity with the mutual influence of the financing level of key competences on the growth of consumer markets.

Non-linear models can explain the emergence of the cyclical development of new products consumer markets created through the introduction of innovative technologies. We consider nonlinear models that reflect the fact of the consumer markets saturation. Indeed, any market has a certain capacity, so its extensions should have limitations that are expressed in the non-linear dependence of the coefficient \( K(t) \). In particular, we can use the following formula for this coefficient

\[
K(t, M(t)) = k_0(t) \left( \frac{1}{1 + M(t)} \right)^\alpha
\]

Here it is supposed that the degree indicator fulfill the condition \( 0 < \alpha < 1 \). In this formula, for the transition coefficient, we can see that for limited values of \( k_0(t) \) and with increasing value of \( M(t) \), this coefficient decreases, which reflects
the marginal utility of the product. A typical form of this coefficient is shown in the following graph (Figure 9).

**Figure 9. Dependence of non-linear coefficient on M(t)**

Consider the numerical modeling of balance variables dynamics, with the use of non-linear coefficient. We use the following initial conditions:

\[
\begin{align*}
I(0) &= 1 \\
M(0) &= 1 \\
L(t) &= 2.5 \\
k_0 &= 2 \\
a &= 0.5
\end{align*}
\]

These values correspond to the stage of rapid growth of financing key competences and, accordingly, to the rapid growth of consumer markets – Figure 10.
The resulting graph shows that at the simulated stage, there is a rapid mutual growth of the balance variables, which, after reaching saturation, goes to stationary values. Obviously, in real economic processes, the stationary regime should also change to further growth (or decline), but since we used the coefficients in the balance equation that are independent of time, we are able to reach stationary values.

However, the results of this simulation demonstrate that with using non-stationary balance coefficients it is possible to obtain a spiral behavior of the dynamics of the balance variables that corresponds to the simulated law of mutual growth of the level of financing key competences and the expansion of new markets. The models under consideration give only a qualitative description of economic processes, without a quantitative picture of these phenomena. A full description of quantitative indicators in the law on creating new markets with the development of key technological competencies is possible with taking into account the specific features and relying on simulation models.

**Conclusion**

The economic law is presented, which is translated into the fact that the development of key technologies leads to the emergence of new consumer markets, resulting in a further increase in funding the development of key competences and the creation of innovative technologies. Thus, there emerges a spiraling mutual development of new technologies and the acquisition of new key competences on the one hand, and the creation and development of new consumer markets on the other hand.
The paper constructs the models the qualitative description of these economic processes. The proposed models are just the first results of the mathematical formalization of the law of mutual influence of the level of competences and the new consumer markets growth. These results open up new possibilities for investigating this interesting phenomenon.

First, further research should include not only qualitative, but also quantitative economic and mathematical models. On the other hand, the derived balance equations can be applied to construct new economic and mathematical models in order to design models for optimal management of the consumer markets growth and the level of key technological competences. There have been derived main balance equations that take into account the effects of delay in the economic processes implementation described by the law of cyclical development of consumer markets.

The law of mutual influence of the level of financing key competences and the new consumer markets growth opens up new opportunities to investigate the most important economic problems of high-tech industries such as competitiveness management and increasing the efficiency of knowledge-intensive industries. Without solving these problems, it is impossible to ensure the effective functioning of the economy in the context of the global economic crisis and the economy restructuring the into a modern knowledge economy (Menshchikova and Sayapin, 2016).

**Acknowledgments**

This paper was financially supported by the Ministry of Education and Science of the Russian Federation on the program to improve the competitiveness of Peoples’ Friendship University (RUDN University) among the world’s leading research and education centers in the 2016-2020.

This paper was financially supported by the Ministry of Education and Science of the Russian Federation on the project No. 26.1146.2017/ПЧ “Development of mathematical methods to forecast efficiency of using space services in the national economy”

**References**


