
Silver in Equity Portfolio Risk Optimization: Polish Investor Perspective

Submitted 14/05/21, 1st revision 12/06/21, 2nd revision 28/07/21, accepted 25/08/21

Izabela Pruchnicka-Grabias¹

Abstract:

Purpose: The study aims to verify whether silver can help reduce the risk of the equity portfolio for a Polish investor without diminishing its return. What are optimal shares of silver and equity in such a portfolio, and does it also improve skewness and kurtosis of the portfolio return distribution compared with a stock portfolio? Do optimal shares change depend on the situation on the silver market?

Design/Methodology/Approach: The author calculates descriptive statistics for silver and equity to decide if the former can diversify assets for the latter. As equity, the WIG20 Total Return index is used. Equity-silver portfolio optimization is conducted in the light of the Markowitz theory. The analysis is done for the period between January 2005 and April 2021.

Findings: The research shows that including silver in an equity portfolio lets both decrease its risk and increase the return during the silver bull market and an extended period, which take part in a growing silver market trend. Conclusions do not change when the short sale is allowed. It is indicated that during silver bear markets 100% equity portfolio is not an efficient portfolio in the light of the Harry Markowitz theory. Simultaneously, the author suggests that although the portfolio diversification conducted in the study results in diminishing risk understood as variance or standard deviation, the results for skewness and kurtosis of created optimum portfolios are various.

Practical Implications: The research may be interesting for institutional and individual Polish investors seeking different diversifying instruments in various market conditions.

Originality/Value: The research contributes to the existing international literature by confirming that silver can be a diversifying asset and indicates its optimal shares in the silver-equity portfolio in different market conditions for a Polish investor endangered with USD/PLN currency rate risk.

Keywords: Silver, precious metals, equity portfolio, Markowitz theory.

JEL codes: G1, F3.

Paper Type: Research Paper.

¹ Collegium of Socio-Economics, Institute of Banking, Warsaw School of Economics, Poland, ipruch@sgh.waw.pl;

1. Introduction

Although plenty of studies have concentrated mainly on gold as a diversifying asset over decades, (Sherman, 1892; Ratner and Klein, 2008; Baur and Lucey, 2010; Beckmann *et al.*, 2015; Arouri *et al.*, 2015; Lucey *et al.*, 2019), silver is usually analyzed together with other precious metals, not as a single commodity. Conclusions present in the literature concerning possible applications of silver are various. Some authors appreciate its low correlation with stock markets, and some others emphasize that it is not as advantageous as gold. Thanks to playing the role of the currency in historical times (Vigne *et al.*, 2017), silver is appreciated by investors as an investment asset. However, it does not play the role of a strategic asset as gold does. (Lau *et al.*, 2017) emphasize that silver has gained lots of investors' attention over the last few years. Portfolio managers use its diversification features. The literature confirms that if used correctly, it can show some haven potential. Therefore, the paper analyses it from the perspective of a Polish equity investor. This is different from the very often studied American market because apart from being a diversifying asset for stocks, silver must also overpower the USD/PLN currency rate.

The purpose of the study is to check whether silver can help reduce the risk of the equity portfolio for a Polish investor without diminishing its return. If so, what are optimal shares of silver and equity in a portfolio? Do optimal shares change depend on the situation on the silver market? Investors' decisions depend only on such variables as expected returns, variances, and covariances, and risk is understood as the variance of returns (Markowitz, 1952). Furthermore, if silver can reduce the portfolio risk without reducing its return, this would mean that the equity portfolio is not efficient considering the Markowitz theory.

After finding optimal portfolios of silver and equity, the author verifies their statistical features based on skewness and kurtosis. Whether the optimal portfolio is only better than the stock portfolio as far as the variance is concerned, can it also bring more good values of the third and fourth central moment?

2. Literature Review

Silver and other precious metals are a subject of the research on hedges against stock markets and other markets, such as currency markets (Sakemto, 2018) or bond markets. (Agyei-Ampomah *et al.*, 2014) apply 13 different bonds and surprisingly find out that silver can be a better hedge for bond markets than gold, although it is not as good as copper.

Different authors check the correlation between precious metals returns and other markets and verify them as hedging or diversifying assets using different methods. Concerning silver and stocks to which the paper is devoted, they instead concentrate on deciding if it can be a part of a diversified equity portfolio or calculate optimal shares of silver in a stock portfolio in different economic conditions. Results of the

research presented in the literature are often different and stay with each other in contradiction. They depend on periods analyzed, countries considered, and methods applied. Therefore, there is a need to expand this field of research for more extended periods and in different countries.

Generally, the literature devoted to silver applications can be divided into two groups. The first one refers to papers in which silver is presented as a good diversification tool for a stock portfolio, either as a hedging asset or a haven. The second group consists of papers showing that it cannot be applied as a diversifying instrument. In the first-mentioned type of papers, there should be mentioned (Hillier *et al.*, 2006) who confirm that silver is low correlated with markets of traditional assets, so they propose it to be a diversification instrument. The authors conclude having analyzed daily data from 1976 to 2004. Silver hedging capability is supposed to be stronger during untypical stock market volatility. They choose the S&P500 index to reflect the behavior of stock markets, so conclusions are made from the American perspective.

Baur and Smales (2020) show silver (apart from gold) as a hedge against geopolitical risk claiming that stock markets are volatile. Jayeola *et al.* (2017) prove that silver can help in portfolio risk reduction, although it is not as efficient as gold. Charles *et al.* (2015) used daily data from January 1977 to October 2013 and claimed that silver and other precious metals are an essential part of an investment portfolio. They show that thanks to the volatile market efficiency, silver returns have been fluctuating over time depending on the economic environment and political factors. Riley (2010) stresses virtues of all precious metals such as advantageous returns and negative correlations with various assets, creating good hedging tools. Hammoudeh *et al.* (2011) check characteristics of silver returns (exactly their dynamics of correlation and volatility) and emphasize that it can be used in risk management. Cochran *et al.* (2012) analyze daily returns for the period 1999 - 2009 and show that silver or other metals and equity returns are subject to the same risk factors. They encourage investors to consider implied volatility of the equity market when metals investments are made. Lucey and Li (2013) examine silver as a haven for stocks and bonds in 1989 - 2013 from the US perspective. They confirm that haven properties of silver change over time. Talbi *et al.* (2020) check the role of precious metals as a haven, hedge, or diversification asset for G-7 countries. Authors summarize that silver offers good diversification benefits and that it is a weak haven for most countries and a strong one for Italy and Germany. Kirkulak-Uludag and Lkhamazhapov (2017) show that silver is a good diversifier for other precious metals as its returns have low correlation coefficients with other precious metals.

Publications from the second group confirm that silver is not an efficient diversification tool. Here, the research conducted by Hood and Malik (2013) should emphasize that silver, contrary to gold, cannot be either a haven or a hedge for stock markets. Authors apply daily data from 1995 to 2010 coming from the American stock market. Sensoy (2013) checks dynamic conditional correlations between returns on precious metals in 1999-2013 and concludes that all of them are generally firmly

correlated, suggesting that diversification among them is limited. The author advises monitoring especially the volatility of silver and gold for hedging. Klein (2017) applies the extended DCC GARCH model in 2000-2017 and examines the dynamic correlation of precious metals with equity markets reflected by stock indexes such as S&P500, FTSE100, EUROSTOXX600, DAX30 and finds out silver to be a haven for stock markets only partially, however, this effect diminishes in time and disappears after 2013. Kayal and Maheswaran (2018) confirm that there exists an explicit dependency between gold and silver volatilities of returns, although they are not the same. They conclude that silver is not a good choice for investments like gold. Dias and Carvalho (2020) reveal that silver cannot act as a haven for portfolio diversification in stock markets in Latin America.

3. Research Methodology

Monthly data from January 2005 to April 2021 are analyzed. The study is done from the perspective of a Polish investor. Silver prices were downloaded in American dollars and recalculated into Polish zloty using close USD/PLN currency rates. All data were used as close prices to avoid random fluctuations and downloaded from the publicly available database www.stooq.com. Stocks are reflected by the WIG20 Total Return index, which is calculated for the 20 most significant and most liquid Polish companies quoted on the Warsaw Stock Exchange.

Contrary to the traditional WIG20 index, it is an income index which means that it comprises both prices of stocks and dividends paid on them. Calculations of descriptive statistics and optimization techniques were proceeded with assessing logarithmic returns on analyzed assets. For optimization, the author assumed two different possibilities. The first one with the short sale of silver and stocks and the second one without using a short sale. It turned out that optimal stock-silver portfolios do not have short-sold assets, which means that conclusions do not depend on applying short sales. Optimization procedure was conducted under the following assumptions:

$$\text{Purpose: } V_p \rightarrow \min \quad (1)$$

Reached by changes of shares: u_1, u_2

Under the following imposed constraints for the first variant (without short sale):

$$u_1 + u_2 = 1 \quad (2)$$

$$u_1, u_2 \geq 0 \quad (3)$$

$$r_p \geq r_{WIG20} \quad (4)$$

And for the second variant (with short sale) with imposed restrictions (2) and (4).

where:

V_p - stock and gold portfolio variance understood as in the (Markowitz 1952; Nowakowski *et al.*, 2002) portfolio theory:

$$V_p = u_1^2 s_1^2 + u_2^2 s_2^2 + 2u_1 u_2 s_1 s_2 \rho_{12} \quad (5)$$

u_1, u_2 - shares of stocks (WIG20 TR index) and silver
 r_p - expected return on portfolio defined as:

$$r_p = u_1 r_1 + u_2 r_2 \quad (6)$$

s_1 - standard deviation on stock returns

s_2 - standard deviation on silver returns

r_1, r_2 - expected returns on stocks and gold

ρ - correlation coefficient between returns on stock and silver

$r_{WIG20,t}$ - return on WIG20 Total Return index.

The data was analyzed in different periods depending on trends on the silver market, that is:

- January 2005 - April 2011 - the increasing trend of silver returns,
- May 2011 - March 2020 - the decreasing trend of silver returns,
- April 2020 - April 2021 - the growing trend of silver returns,
- January 2005 - April 2021 covering the whole researched period that is both bull and bear markets.

In each of the cases mentioned above, it is assumed that an investor creates a stock-silver portfolio at the beginning of the analyzed period to take advantage of silver as a diversifier. Skewness and kurtosis for single assets and portfolios are understood as given below. Skewness shows the level of asymmetry of the distribution of rates of return around the mean. If it is positive, it induces more significant profits and more minor losses than for the normal distribution for which skewness is zero. Mathematically skewness is presented as (Lhabitant, 2006: 436-437):

$$s = \frac{T}{(T-1)(T-2)} \sum_{t=1}^T \left(\frac{r_t - r_{av}}{s} \right)^3 \quad (7)$$

Kurtosis shows the scale of distribution tails. The higher it is, the riskier the asset is. Its value for a normal distribution is zero. Mathematically kurtosis is equal to (Lhabitant, 2006: 437):

$$k = \frac{T(T+1)}{(T-1)(T-2)(T-3)} \sum_{t=1}^T \left(\frac{r_t - r_{av}}{s} \right)^4 \quad (8)$$

where:

T - number of returns

r_{av} - average return on the analyzed asset

r_t - monthly return

s - standard deviation on the analyzed asset.

4. Results and Discussion

Table 1 presents Pearson correlation coefficients and covariances between WIG20 Total Return index and silver. The gathered data suggest that correlations are relatively low in all periods, and they are not always significant. This is a desired feature for diversification. Such results confirm that silver can be used as a diversifying asset for the equity portfolio in any period out of analyzed ones. Therefore, the author may further study checking if silver can help reduce risk in the equity portfolio without diminishing its return.

Table 1. Relations between rates of return on the WIG20 Total Return index and silver.

	January 2005 - April 2011	May 2011 - March 2020	April 2020 - April 2021	January 2005 - April 2021
Pearson correlation coefficients	0,1924 p=0,0981	0,3179 p=0,0008	0,1573 p=0,6078	0,2571 p=0,0003
Covariance	14,0291	13,0607	14,4875	14,9827

Source: Own calculations.

Data depicted in Tables 2 and 3 show that silver has both a higher rate of return and higher variance for three periods. Only in one period during the bear market on silver (May 2011-March 2020), both return and variance were worse for silver than for stocks. It means that silver is not always advised as a diversification tool. Most of all, it helps improve the stock portfolio in the long period when some of the time the market rises, and some diminishes or when the silver market grows. From January 2005 to April 2011, the WIG20 Total Return index achieved the average monthly rate of return equal to 0,91% and the variance of 56,32. At the same time, silver's return was higher (2,62%) with also higher variance (94,37). From May 2011 - March 2020, the average return on stocks was -0,28%, whereas silver was -1,15%.

As far as the variance is concerned, for equity, it was 25,33, and for silver, 66,65. Such numbers mean that some part of the stock portfolio can be replaced with silver from the point of view of the Markowitz portfolio theory as the rational investor does not agree for both lower returns and higher risk. Such a portfolio with stocks only would not be the efficient one. A different situation is observed in April 2020 to April 2021, where WIG20's average monthly return is 2,34% and silver's 4,73%, whereas variance is 54,70 and 155,07, respectively. In this case, silver may be a recommended asset in the portfolio, verified by an optimization technique. The last period analyzed is the whole time taken under consideration from January 2005 to April 2021. Stock's return is 0,35% and silver's one is higher (0,69%). As for variances, it is lower for stocks (39,31) than for silver (86,41).

Moving on to skewness and kurtosis, for the period January 2005 - April 2011 WIG20 Total Return index has more negative skewness (-0,78) than silver (-0,36), which means that including silver in the portfolio may improve the third central moment of the distribution. The same can be concluded for kurtosis, which is higher for stocks

than for silver, which means that silver is a desired instrument in the portfolio from this point of view. Contrary conclusions can be drawn for the period May 2011 - March 2020. Here WIG20's skewness (-0,28) is less harmful than for silver (-0,35), whereas WIG20 has lower kurtosis (3,30) than silver (4,47).

Thus, considering skewness and kurtosis here, silver does not seem to be the asset that lets us decrease the portfolio's risk level. So, returns and all risk measures such as standard deviation, skewness, and kurtosis lead to the conclusion of silver being not helpful in this period as a diversifying asset. The next analyzed time is April 2020 - April 2021. It is different both from January 2005 - April 2011 and May 2011 - March 2020. In this case, skewness for stocks is much higher (54,70) than for silver (0,07) which is a desired feature which means that favorable rates of return are more probable for the WIG20 index than for silver. It looks differently for kurtosis, where higher kurtosis is observed for WIG20 (3,71) than for silver (2,86), and in this case, it means that the risk of untypical rates of return (far away from the normal distribution) is higher for stocks which are not desired.

Table 2. Descriptive statistics for the WIG20 Total Return index in analyzed periods.

January 2005 - April 2011				
	Percentiles	Smallest		
1%	-26,68283	-26,68283		
5%	-11,75939	-15,76127		
10%	-8,549348	-15,02138		
25%	-3,645062	-11,75939	Number observations of	75
50%	2,028384	Largest	Mean	0,9088242
75%	5,839097	11,05636	Standard deviation	7,504475
90%	9,67238	11,91692	Variance	56,31714
95%	11,05636	15,77423	Skewness	-0,7800617
99%	17,36186	17,36186	Kurtosis	4,473825
May 2011 - March 2020				
	Percentiles	Smallest		
1%	-15,51997	-15,63744		
5%	-7,70927	-15,51997		
10%	-6,432178	-10,00272		
25%	-3,555483	-9,841072	Number observations of	107
50%	-0,7001107	Largest	Mean	-0,2838015
75%	3,692889	8,596242	Standard deviation	5,032541
90%	6,330666	8,831839	Variance	25,32647
95%	8,023407	9,09178	Skewness	-0,277761
99%	9,09178	9,960147	Kurtosis	3,304202
April 2020 - April 2021				
	Percentiles	Smallest		
1%	-12,02981	-12,02981		
5%	-12,02981	-4,929595		
10%	-4,929595	-2,112766		
25%	-1,829639	-1,829639	Number observations of	13
50%	1,831694	Largest	Mean	2,336729
75%	4,967347	4,967347	Standard deviation	7,396048
90%	8,717142	8,076169	Variance	54,70153
95%	18,82799	8,717142	Skewness	54,70153
99%	18,82799	18,82799	Kurtosis	3,706709
January 2005 - April 2021				
	Percentiles	Smallest	195	
1%	-15,76127	-26,68283		
5%	-10,00272	-15,76127		

10%	-7,051842	-15,63743		
25%	-3,517927	-15,51997	Number of observations	195
50%	0,4776935	Largest	Mean	0,3496028
75%	4,444873	11,91691	Standard deviation	6,269797
90%	8,023407	15,77423	Variance	39,31035
95%	9,67238	17,36186	Skewness	-0,4325628
99%	17,36186	18,82799	Kurtosis	4,621073

Source: Own calculations.

From January 2005 - April 2021, both skewness and kurtosis are worse for stocks than for silver. To be exact, skewness for WIG20 is -0,43 and for silver -0,12 while kurtosis for WIG20 is 4,62 and silver is lower (3,75). It means that in an extended period, silver seems to be an efficient equity portfolio diversification tool no matter if it is analyzed from the point of view of the Markowitz theory where the second central moment of the distribution is applied or if an investor considers third and fourth central moments of the distribution.

Table 3. Descriptive statistics for silver in analyzed periods.

January 2005 - April 2011				
	Percentiles	Smallest		
1%	-26,73966	-26,73966		
5%	-13,57831	-20,19646		
10%	-9,547221	-13,87682		
25%	-3,704127	-13,57831	Number of observations	75
50%	3,326831	Largest	Mean	2,618981
75%	9,475946	16,91346	Standard deviation	9,714685
90%	13,34468	19,09002	Variance	94,37511
95%	16,91346	22,81271	Skewness	-0,3626264
99%	24,07257	24,07257	Kurtosis	3,201623
May 2011 - March 2020				
	Percentiles	Smallest		
1%	-21,89258	-32,73139		
5%	-13,78993	-21,89258		
10%	-9,984164	-17,31221		
25%	-6,268593	-16,48962	Number of observations	107
50%	-1,326504	Largest	Mean	-1,151252
75%	3,644025	14,33599	Standard deviation	8,163993
90%	9,616476	15,64694	Variance	66,65078
95%	13,16084	16,70812	Skewness	-0,3501709
99%	16,70812	17,45977	Kurtosis	4,467237
April 2020 - April 2021				
	Percentiles	Smallest		
1%	-19,17475	-19,17475		
5%	-19,17475	-8,379109		
10%	-8,379109	-4,408666		
25%	-1,346822	-1,346822	Number of observations	13
50%	1,955071	Largest	Mean	4,735379
75%	14,5893	14,5893	Standard deviation	12,45274
90%	17,71168	15,36708	Variance	155,0706
95%	28,98775	17,71168	Skewness	0,0733758
99%	28,98775	28,98775	Kurtosis	2,858872
January 2005 - April 2021				
	Percentiles	Smallest		
1%	-26,73966	-32,73139		

5%	-13,78993	-26,73966		
10%	-9,809933	-21,89258		
25%	-5,249018	-20,19646	Number of observations	195
50%	0,632437	Largest	Mean	0,6912799
75%	6,439137	19,09002	Standard deviation	9,29587
90%	12,80952	22,81271	Variance	86,41319
95%	15,64694	24,07257	Skewness	-0,1175225
99%	24,07257	28,98775	Kurtosis	3,74792

Source: Own calculations.

The conducted optimization shows different results in different periods. Therefore, optimal portfolio structure depends on the trend in the silver market. From January 2005 to April 2011, the optimal share of silver in the equity portfolio is 34%. From May 2011 to March 2020, it is not advised to apply silver if an investor aims at constructing the optimal portfolio. In April 2020 to April 2021, the optimum share of silver is 22%, and in January 2005 to April 2021, it is 27%. Therefore, silver should be used in the equity portfolio as a diversifying asset when an investor expects a rising trend on the silver market. It should be emphasized that results do not depend on using the short sale of stocks or silver by an investor. Thus, the optimum portfolio in any period is the one without any short-sold assets.

Table 4. Results of portfolio risk optimization for the WIG20 Total Return index and silver.

	January 2005 - April 2011	May 2011 - March 2020	April 2020 - April 2021	January 2005 - April 2021
Silver share	0,34	1	0,22	0,27
Stocks share	0,66	0	0,78	0,73
Stocks average monthly rate of return	0,91	-0,28	2,34	0,35
Silver average monthly rate of return	2,62	-1,15	4,73	0,69
Assumed portfolio rate of return	>=WIG20 Total Return index	>=WIG20 Total Return index	>=WIG20 Total Return index	>=WIG20 Total Return index
Portfolio rate of return	1,49	-0,28	2,86	0,44
Silver variance	94,37	66,65	155,07	86,41
WIG20 Total Return index variance	56,32	25,33	54,70	39,31
Portfolio variance	41,74	25,33	45,76	33,15

Source: Own calculations.

In the next step, the author creates optimized portfolios with silver and stocks for periods in which it turned out to be desired, checks such new portfolios characteristics, and compares with equity-only portfolios.

Comparison of data presented in table 5 with these depicted in Tables 2 and 3 leads to the conclusion that silver is an excellent diversifying asset for the equity portfolio in the light of the (Markowitz, 1952) theory. Compared with a 100% stock portfolio, silver lets reduce risk measured with variance and increase the return during three out of four analyzed periods. It means that portfolios are not efficient portfolios during these periods; they can improve their characteristics for the risk and return. It does not work only during the silver bear market. From January 2005 to April 2011, the return

on equity is 0,91%, whereas, on the silver-equity portfolio, it is 1,49%. As far as the variance is concerned, it is lower for the portfolio (41,74) than equity (56,32). From May 2011 to March 2020, there existed a decreasing trend on the silver market, which meant that the stock portfolio was efficient, if an investor can only choose between silver and stocks. From April 2020 to April 2021, the return on stocks is 2,24%, and it can be increased by adding 22% of silver to the portfolio to 2,86%. It also helps make variance lower from 54,70 to 45,76.

Table 5. Descriptive statistics for optimal portfolios built of silver and stocks from WIG20 Total Return index in analyzed periods.

January 2005 - April 2011				
	Percentiles	Smallest		
1%	-24,47746	-24,47746		
5%	-9,738139	-12,92866		
10%	-5,913619	-10,52865		
25%	-3,015418	-9,738139	Number observations	of 75
50%	2,389593	Largest	Mean	1,490278
75%	6,30546	9,916936	Standard deviation	6,460479
90%	8,840141	10,40251	Variance	41,73778
95%	9,916936	11,27681	Skewness	-1,012372
99%	13,04777	13,04777	Kurtosis	5,035752
May 2011 - March 2020				
	Percentiles	Smallest		
1%	-21,89258	-32,73139		
5%	-13,78993	-21,89258		
10%	-9,984164	-17,31221		
25%	-6,268593	-16,48962	Number observations	of 107
50%	-1,326504	Largest	Mean	-1,151252
75%	3,644025	14,33599	Standard deviation	8,163993
90%	9,616476	15,64694	Variance	66,65078
95%	13,16084	16,70812	Skewness	-0,3501709
99%	16,70812	17,45977	Kurtosis	4,467237
April 2020 - April 2021				
	Percentiles	Smallest		
1%	-8,989047	-8,989047		
5%	-8,989047	-8,063529		
10%	-8,063529	-1,944258		
25%	-0,9970031	-0,9970031	Number observations	of 13
50%	4,638367	Largest	Mean	2,864432
75%	7,319907	7,319907	Standard deviation	6,764461
90%	9,680169	8,294538	Variance	45,75793
95%	13,71593	9,680169	Skewness	-0,3504935
99%	13,71593	13,71593	Kurtosis	2,263868
January 2005 - April 2021				
	Percentiles	Smallest		
1%	-16,13946	-24,93151		
5%	-9,183397	-16,13946		
10%	-6,147959	-16,08962		
25%	-2,954979	-13,53044	Number observations	of 195
50%	0,5141947	Largest	Mean	0,4418556
75%	4,905586	11,39934	Standard deviation	5,757967
90%	7,101633	12,20275	Variance	33,15418
95%	9,410514	12,5541	Skewness	-0,6099151
99%	12,5541	12,63778	Kurtosis	4,407211

Source: Own calculations.

The last period analyzed is the whole time from January 2005 to April 2021. Results show that the equity portfolio's return is 0,35% while the silver-equity ones are 0,44%. Variance also gives better results for a diversified portfolio (33,15 to 39,31 for the 100% stock portfolio). So, in short, stock portfolio diversification with silver lets both increase the return and decrease risk measured with variance. However, measuring risk with skewness and kurtosis gives different conclusions. Skewness is worse for all three periods in which variance is lower for a diversified portfolio and indicated higher risk. Kurtosis increased for a diversified portfolio in January 2005 - April 2011 from 4,47 to 5,03, and it dominates for April 2020 - April 2021 from 3,71 to 2,26 and for January 2005 to April 2021 from 4,62 to 4,41. It suggests that a portfolio optimized against skewness and kurtosis might require different shares.

5. Conclusion

It is shown that silver has a low correlation of rates of return with returns on equity. Thus, it may be a desired equity portfolio diversifying asset. However, its applicability depends on the silver market trend. As shown, silver can reduce the stock portfolio risk while increasing its return. This, in turn, means that a 100% stock portfolio is not the efficient portfolio understood according to the concept derived by (Markowitz, 1952) when the silver market rises. To be exact, silver portfolio diversification can be applied when the silver market is expected to grow, or the investment is made for a very long period when grows are highly probable for some of it. Silver shares in the equity portfolio are different in different periods, starting during the silver bull market from 22% of the whole portfolio and ending with 34% of it. Conclusions are the same no matter if an investor is allowed to use the short sale of silver or stocks or not. However, although the conducted portfolio diversification decreased risk measured with variance or standard deviation, results for skewness and kurtosis are various.

The research may be interesting both for institutional and individual Polish investors who seek different diversifying instruments. Investments in silver can be made both directly when buying bars and coins or by using derivatives. In the second case, investors must also consider possible differences between derivatives prices on the silver and silver cash market, which may appear. As far as the WIG20 Total Return index is concerned, it can be invested directly by buying stocks proportionally to their share in this index. Otherwise, WIG20 could be used, and in such a case, futures contracts can be applied, which are liquid instruments on the Warsaw Stock Exchange. One can also apply CFD contracts on WIG20.

However, the difference between WIG20 and WIG20 Total Return is that the second also considers dividends paid by companies, so its rates of return are more advantageous than for traditional WIG20.

The added value of the paper is a very long research period, the maximum one for which quotations of the WIG20 Total Return index are available. The research is conducted during different market conditions separately, both for silver bull markets

and bear markets. Besides, there not many studies analyzing silver as a single asset of portfolio diversification. It is usually a part of the research devoted to gold mainly. Another advantage is analyzing from the perspective of a Polish investor, which also considers the currency rate risk. Numerous studies conducted for American investors do not consider USD/PLN risk.

Further research can concentrate on applying theories other than the Markowitz one to compare results. Possibilities of further studies may also include creating a diversified portfolio of three assets like, for example, stocks, bonds, and silver, as well as optimizing other risk measures.

References:

- Agyei-Ampomah, S., Gounopoulos, D., Mazouz, K. 2014. Does gold offer a better protection against losses in sovereign debt bonds than other metals? *Journal of Banking & Finance*, 40, 507-521.
- Arouri, M.E.H., Lahiani, A., Nguyen, D.K. 2015. World gold prices and stock returns in China: Insights for hedging and diversification strategies. *Economic Modelling*, 44, 273-282.
- Baur, D.G., Lucey, B.M. 2010. Is gold a hedge or a heaven? An analysis of stocks, bonds and gold, *Financial Review*, 45, 217-229.
- Baur, D.G., Smales, L.A. 2020. Hedging geopolitical risk with precious metals, *Journal of Banking and Finance*, Elsevier, 117(C).
- Beckmann, J., Berger, T., Czudaj, R. 2014. Does Gold Act as a Hedge or a Safe Haven for Stocks? A Smooth Transition Approach, *Economic Modelling*, 48, 16-24.
- Charles, A., Darne, O., Kim J.H. 2015. Will precious metals shine? A market efficiency perspective. *International Review of Financial Analysis*, 41, 284-291.
- Cochran, S.J., Mansur, I., Odusami, B. 2012. Volatility persistence in metal returns: A FIGARCH approach, *Journal of Economics and Business*, 64(4), 287-305.
- Dias, R., Carvalho, L.C. 2020. Hedges and safe havens: an examination of stocks, gold, and silver in Latin America's stock market. *Brazilian Journal of Management*, 13(5), 1114 -1132.
- Hammoudeh, S., Malik, F., McAleerc, M. 2011. Risk management of precious metals. *The Quarterly Review of Economics and Finance*, 51(4), 435-441.
- Hillier, D., Draper, P., Faff, R. 2006. Do Precious Metals Shine? An Investment Perspective. *Financial Analysts Journal*, 62(2), 98-106.
- Hood, M., Malik, F. 2013. Is gold the best hedge and a heaven under changing stock market volatility? *Review of Financial Economics*, 22(2), 47-52.
- Jayeola, D, Zulhaimy, I., Sufahani, S.F. 2017. Effects of diversification of assets in optimizing risk of portfolio, *Malaysian Journal of Fundamental and Applied Sciences*, 13(4), 584-587.
- Kayal, P., Maheswaran, S. 2018. A study of excess volatility of gold and silver, IIMA-IGPC Conference on Gold and Gold Markets. India Gold Policy Centre, 1-34.
- Kirkulak-Uludag, B., Lkhamazhapov, Z. 2017. Volatility dynamics of precious metals: evidence from Russia. *Czech Journal of Economics and Finance*, 67(4), 300-317.
- Klein, T. 2017. Dynamic correlation of precious metals and flight-to-quality in developed markets. *Finance Research Letters*, 23, 283-290.

- Lau, M.C.K., Vigne, S.A., Wang, S., Yarovaya, L. 2017. Return spillovers between white metals ETFs: the role of oil, gold, and global equity. *International Review of Financial Analysis*, 52, 316-332.
- Lhabitant, F.S. 2006. *Handbook of hedge funds*, John Wiley & Sons, Ltd., Chichester, 436-437.
- Lucey, B., Li, S. 2015. What precious metals act as safe havens, and when? Some US evidence. *Applied Economic Letters*, 22(1), 35-45.
- Lucey, B.M., Peat, M., Sević, A., Vigne, S.A. 2019. What is the optimal weight for gold in a portfolio? *Annals of Operations Research*. Springer.
- Markovitz, H. 1952. Portfolio selection. *The Journal of Finance*, 7(1), 77-91.
- Nowakowski, J., Niedziółka P., Mieloszyk J. 2002. Portfel inwestycyjny banku (Bank investment portfolio). *Difin*, 155-156.
- Ratner, M., Klein, S. 2008, The portfolio implications of gold investment. *Journal of Investing*, 17, 77-87.
- Riley, C. 2010. A new gold rush: Investing in precious metals. *Journal of Investing*, 19, 95-100.
- Sakemoto, R. 2018. Do precious and industrial metals act as hedges and safe havens for currency portfolios? *Finance Research Letters*, 24, 256-262.
- Sensoy, A. 2013. Dynamic relationship between precious metals. *Resources Policy*, 38(4), 504-511.
- Sherman, E.J. 1892. Gold: a conservative, prudent diversifier. *The Journal of Portfolio Management*, 8, 21-27.
- Stooq.com. 2021. Retrieved from: www.stooq.com.
- Talbi, M., Bedoui, R., de Peretti, C., Belkacem, L. 2020. Is the role of precious metals as precious as they are? Revisiting the role of precious metals for the G-7 stock markets: A multivariate vine copula and BiVaR approaches. HAL archives ouvertes, fffal-01664146, 1-4. Retrieved from: <https://hal.archives-ouvertes.fr/hal-01664146/document>.
- Vigne, S.A., Lucey, B.M., O'Connor, F.A., Yarovayad, L. 2017. The financial economics of white precious metals - A Survey *International Review of Financial Analysis*, 52(C), 292-308.