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The Equilibrium Value of The Euro/$ US Exchange Rate: An Evaluation of Research

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Abstract

The ultimate object of research concerning the Euro is to answer the following questions: (#1) What is the equilibrium trajectory of the nominal euro, measured as dollars/euro? (#2) To what extent has the equilibrium nominal euro been determined by relative prices (PPP), and to what extent has it been determined by real fundamentals? (#3) How important have been the transitory factors in affecting the value of the euro? (#4) Is the euro currently undervalued, and by what criteria? Our evaluation of recent research concerning the answers to these questions, is the subject of this paper.

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1.1. Why it is important to know the equilibrium value of the exchange rate

There have been several notable studies concerning the equilibrium real value of the Euro. The first set was delivered at a joint European Central Bank[ECB]/Deutsche Bundesbank conference in March 2000, a second set consists of two studies by the staff of the European Central Bank, a third set was presented at a conference at La Banque de France in June 2000, a fourth study was done at the Ministry of Finance of France, and a fifth set consisted of pa-

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pers written at academic institutions: the Sorbonne -Université Paris I, CEFI: Université de la Méditerranée, CIDEI: La Sapienza, University of Rome and at EHSAL in Brussels. The aim of this article is to synthesize/evaluate their results[2] to answer the question: what have been the determinants of the equilibrium real value of a synthetic Euro.

In all cases, the researchers constructed a synthetic Euro exchange rate. The hypothesis is that a valid theory concerning the actual real value euro, whose birth was only a few years ago, should be able to explain the real value of the synthetic euro based upon many years of data. The advent of the ECB can be expected to change monetary policy and relative prices, but monetary policy should not affect the determination of the longer-run equilibrium real value of the euro.

The equilibrium value of the real exchange rate is a sustainable rate that satisfies several criteria. First; it is consistent with internal balance. This is a situation where the rate of capacity utilization is at its longer run stationary mean[3]. Second, it is consistent with external balance. The latter is a situation where, at the given exchange rate, investors are indifferent between holding domestic or foreign assets. At the equilibrium real exchange rate, there is no reason for the exchange rate to appreciate or depreciate. Hence, portfolio balance or external balance implies that real interest rates between the two countries should converge to a stationary mean. As long as there are current account deficits, the foreign debt and associated interest payments rise. If the current account deficit/foreign debt exceeds the growth rate of real GDP, then the ratio of the debt/GDP and the burden of the debt - net interest payments/GDP - will rise. When the debt burden is sufficiently high, devaluation will be required to earn enough foreign exchange

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[2] Other pertinent studies are cited in the references contained in the papers evaluated here.

[3] This is a more precise concept than is the NAIRU.

through the trade balance to meet the interest payments. The condition for external equilibrium in the longer run is that the ratio of the foreign debt/GDP stabilizes at a tolerable level.

Define “misalignment” as the deviation of the actual real exchange rate R(t) from its equilibrium value. Any derived equilibrium real exchange rate must be an attractor: the actual real exchange rate converges to the equilibrium rate[4]. The convergence can be produced either by changes in the nominal exchange rate or by changes in relative prices[5].

The return of the UK to the gold standard in 1925, the exchange rates established at the Bretton Woods conference, the exchange rate of 1 Ost- mark for 1 D-Mark with German unification are examples where the pegged exchange rates were not consistent with internal balance. These exchange rates were not sustainable: they were overvalued, and the tradable sectors lost their competitivity. Governments may try to achieve internal balance at an overvalued exchange rate by trying to lower interest rates, and stimulate domestic demand to offset the decline in the trade balance. In that case, external balance/portfolio balance condition would be violated. Investors would try to exchange domestic assets for foreign assets yielding a higher return. The exchange rate would then tend to depreciate. Hence, the initial exchange rate would not be sustainable.

There are several reasons why the ECB's monetary policy, which aims to "stabilize" the price level, must be conditioned upon a concept of the equilibrium real exchange rate[6]. *First*: if the nominal exchange rate is depreciating the ECB should like to know the reason why. If the equilibrium real rate has not changed then a depreciation of the nominal rate can be expected to lead to more inflation.

4 The equilibrium rate may be a distribution, as occurs in stochastic control models.

5 Stein and Paladino (1999) explain the currency crises in this way.

6 See Issing (2000) for an extremely thoughtful discussion of the viability of the monetary union.

In that case, the monetary policy should be reexamined. If the nominal depreciation was produced by a depreciation of the equilibrium real rate, one should not necessarily expect more inflation. Monetary policy need not necessarily be tightened. *Second*: The question then becomes: what has produced the decline in the equilibrium real rate? the ECB should know if there can there be internal balance at the given real exchange rate, when the interest rates in the Euro area are equal to those in the US? The answer to this question is important in the formulation of interest rate policy that is consistent with a "satisfactory" rate of capacity utilization.

Third: The EC is in the process of expanding its membership. An important question is: what will be the effect upon the equilibrium real value of the Euro by adding members to the monetary union? Norms of fiscal policy - the ratio of budget deficits/GDP - have been promulgated both for current and for new members of the EU. One should know what are the effects of different fiscal policies upon the equilibrium exchange rate of the Euro. The conventional Mundell- Fleming theory claims that an expansionary fiscal policy, especially if it is associated with a contractionary monetary policy, leads to exchange rate appreciation. The NATREX model discussed below claims that the Mundell-Fleming hypothesis is correct in the medium run, but it is more than reversed in the longer run. The equilibrium real exchange rate will depreciate in the longer run below its initial value. Consequently, the ECB should have both a theory and evidence concerning the mechanism linking fiscal policy to the exchange rate in the medium to the longer run.

1.2. Organization

In all the studies evaluated here, the researchers constructed a synthetic Euro exchange rate. The hypothesis is that a valid theory concerning the actual real value euro, whose birth was only a few years ago, should be able to explain the *real* value of the synthetic euro based upon many years of data. The nominal exchange rate is

N(t) = dollars/Euro, where a rise is an appreciation of the Euro. The real exchange rate R(t) of the Euro, where a rise is an appreciation of the real synthetic Euro relative to the $US, can be defined in several ways. Generally, the researchers use equation (1), where the ratio p(t)/p*(t) is the Euro/foreign GDP price deflators[7]. The period covered is either 1973:1 - 2000:1 or 1948:1 - 2000:1.

$$R(t) = N(t)p(t)/p^*(t) \quad (1)$$

The ECB researchers divided the world into two blocs; the US, UK, Japan and Switzerland and the Euro bloc consisting of a weighted average of the eleven countries that currently comprise the Euro area. Liliane Crouhy-Veyrac considered the US vis-a-vis a weighted average of the Euro-11. Johan Verrue considered two areas: the US and a weighted average of the four largest countries of the EU - Germany, France, Italy and Spain. Romain Duval and Laurent Maurin related the US to a weighted average of the Euro-3: France, Germany and Italy. Clostermann and Schnatz calculated a geometrically weighted average of the dollar exchange rates of the individual EMU countries, where the weights are derived from the BIS. Since we have Crouhy-Veyrac's data, we shall use them as a basis for our presentation[8].

Figure 1 graphs the two exchange rates: the nominal N(t), and the real R(t) value of the euro as four quarter moving average (MA). A rise is an appreciation of the Euro. Since 1985, the two measures of the real and nominal synthetic euro are almost identical. From 1978 - 85, their trends were similar though their "levels" were dif-

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[7] Some researchers use labor costs instead of broad based indexes. There are advantages and disadvantages to each measure. See, for example, Clostermann and Friedmann (1998). Crouhy-Veyrac shows that the two measures of the real value of the euro relative to the $US, based upon GDP deflators or wage deflators, have been almost identical since 1980.

[8] France, Germany and Italy account for over 70% of the synthetic Euro, so Verrue's synthetic Euro should be close to that estimated by the others. In fact, Clostermann and Schnatz showed that the real value of the synthetic Euro and the real value of the DM moved very closely together.

ferent. We can see that the large variations in the nominal rate were not the result of a relatively constant real rate and large variations in relative prices.

Figure 1. Real R(t) = EUUSREDPMA, Nominal N(t) = EUUSNERMA 4Q moving averages. Rise is an appreciation of euro.

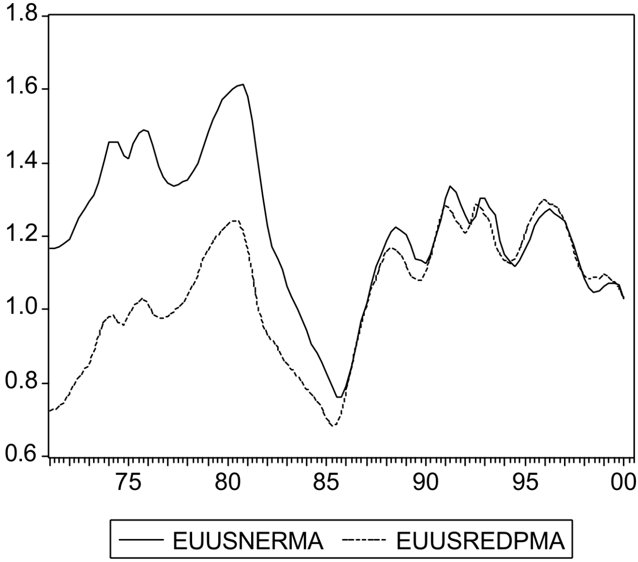

Figure 1

The real value of the euro relative to the \$US:
R(t) = N(t)p(t)/p(t) = EUUSREDPMA,*
and nominal value of the euro relative to the \$US:
N(t) = \$US/Euro= EUUSNERMA.
A rise is an appreciation of the euro. MA= 4Q moving average

The researchers carefully examined the literature concerning the determination of exchange rates, in order to evaluate the explanatory powers of the various techniques, models and hypotheses. They discarded those models that were: (a) non-operational, in the sense that the crucial variables were not objectively measurable, or (b) whose structural equations have been shown to be inconsistent with the evidence. They ended up by going in two directions. In one direction, they took an empirical/econometric approach that is not model specific. In the other direction they used a theoretical model that implies econometric equations. The former may be grouped under the heading BEER, behavioral equilibrium real exchange rate[9], and the latter takes as a point of departure the NATREX model. Makrydakis, de Lima, Claessens and Kramer [ECB: M] describe the alternative approaches as follows.

"The BEER, unlike the ...NATREX approaches that rely on a structural equilibrium concept, is based upon a statistical notion of equilibrium...[BEER] attempts to explain the actual behaviour of the real exchange rate in terms of a set of relevant explanatory variables, the so called 'fundamentals'. The fundamental exchange rate determinants are selected according to what economic theory prescribes as variables that have a role to play over the medium to shortterm....[In BEER]… the underlying theoretical model does not have to be specified. The exchange rate equilibrium path is then estimated by quantifying the impact of the 'fundamentals' on the exchange rate through econometric estimation of the resultant reduced form."

Both approaches are positive, and not normative, economics. There is no welfare significance, or value judgments, implicit in the derived equilibrium real exchange rate. There is no implication that exchange rates should be managed. The principal difference between the BEER and the NATREX, is that the NATREX takes as its point of departure a specific theoretical dynamic stock-flow model to arrive at a reduced form where the equilibrium real exchange rate depends upon relative thrift and relative productivity differences. The papers by [ECB:M] and Clostermann and Schnatz [C-S] take the BEER approach with the Euro. The D-Mark generally has a weight of 37% in the synthetic euro. Clostermann and Schnatz show that the real values of the synthetic euro and the D-Mark move very closely together during the period 1975-99, with a correlation coefficient of 0.98. I therefore also include the papers by

[9] The BEER approach is based upon Clark and MacDonald (1999).

Clostermann and Friedmann[10] (1998) and by Clark and MacDonald (1999) who use a BEER aproach for the D-Mark.

In part 2 the BEER results are evaluated, and are compared in summary table 1. The papers by Detken, Dieppe, Henry, Marin and Smets [ECB: D], Crouhy-Veyrac, Duval, Ministry of Finance of France, Maurin, Gandolfo and Felettigh[11] and by Verrue use the NATREX approach to estimate the "equilibrium" real value of the Euro. Part 3 is a brief exposition of the NATREX model that is used by these authors, and discusses the empirical results of [ECB:D] who examine the structural equations. In part 4, we compare the papers that examine the implied reduced form equations. The results are summarized in table 2. In part 5, we examine the implications for the equilibrium *nominal* value of the euro.

2. The Behavioral Equilibrium Exchange Rate (BEER)

The "B" for "behavioral" in BEER means that there is no explicit underlying structural model. It is exclusively a quest for a cointegrating equation for the real exchange rate. There are differences in the approaches and results in the various papers, but I shall try to present them in terms of their common characteristics.

The authors generally have in mind the requirements of internal/external balance. The internal balance requirement is equation (2). Evaluated at capacity output: investment I less saving S plus the current account CA must be zero. Let u(t) denote the ouput gap, or the deviation of the actual rate of capacity utilization from its stationary mean.

$$I(t) - S(t) + CA(t) = 0, \quad u(t) = 0. \qquad (2)$$

[10] They are with the Bundesbank and have written a series of papers on the real value of the DMark.

[11] See Gandolfo's forthcoming book on international finance for a masterful evaluation of the subject. Here, we omit a discussion of Gandolfo and Felittigh study of the euro due to its econometric complexity.

The equilibrium real exchange rate affects the current account and investment. A sustainable rate must be consistent with equation (2). The variables, vector Z(t), that determine the components of these functions are referred to as the real fundamentals. Denote the equilibrium real exchange rate R[Z(t)] to indicate that it depends upon the real fundamentals Z(t). All of the researchers reject the hypothesis that the real equilibrium exchange rate is a constant, as is claimed by the theory of Purchasing Power Parity (PPP). See figure 1 above. Moreover, the researchers reject the monetary models with PPP, which have been very popular from the 1970s to the mid 1990s[12]. The quest is for a cointegrating equation for the real fundamentals Z(t), that explain in an econometric sense the long-run value of the real exchange rate.

The external balance/portfolio balance requirement varies among the studies. Most of the empirical/econometric studies use equation (3), the uncovered interest rate parity over a long horizon. The expectation of the appreciation of the real exchange rate over a medium run horizon, is proportional to the foreign r*(t) less the domestic r(t) real long-term interest rate. The longer period is used because it is well known that the uncovered interest rate parity theory/rational expectations are rejected when short period rates are used. The error term e(t) reflects the difference between the mathematical expectation of the equilibrium exchange rate and its actual value.

The equilibrium real rate R[Z(t)] is obtained from a solution of equation (2). The empirical equation for the real exchange rate R(t) is equation (3).

$$R(t) = R[Z(t)] + h[r(t) - r^*(t)] + e(t), \; h > 0 \qquad (3)$$

This equation links the longer run R[Z(t)] and the "shorter" run h[r(t) - r*(t)] to the actual real exchange rate R(t). The researchers generally use a VEC to allow for a lagged adjustment of the actual

[12] See the volume edited by MacDonald and Stein (1999) for a discussion of what we know and what we do not know about equilibrium exchange rates, and Stein and Paladino (1997) for an evaluation of alternative theoretical approaches.

rate to the equilibrium rate. This may be due a slow convergence of u(t) to zero.

We may summarize the empirical/econometric BEER studies concerning the equilibrium value of the synthetic Euro as the search for cointegrating equation R(t) = BZ(t), where Z(t) are longer run real fundamentals, and e(t) is stationary with a zero mean. One claims that R(t) converges to the equilibrium BZ(t). The techniques involve VEC analysis, the examination of whether the coefficients have the hypothesized signs and if the only variable that is weakly exogenous is the real exchange rate[13]. The studies differ in what are the elements in vector Z(t) of the exogenous fundamentals.

2.1 Empirical/Econometric: The Behavioral Equilibrium Exchange Rate

Table 1 compares four studies that use the BEER approach in terms of their common characteristics. All the studies agree that there are real variables that can produce a cointegration equation with the real exchange rate. Each cointegrating equation passes the usual econometric tests and does track the real value of the synthetic Euro and the real value of the DMark. Clostermann and Schnatz [C-S] show that their equation for R[Z(t)] outperforms a random walk and the superiority improves as the horizon increases. The real value of the Euro/$US is not a stationary, *constant mean reverting*, variable. This is another demonstration of the economic limitations of the PPP hypothesis.

Six variables, the rows in table 1, are considered as possible fundamentals Z(t) in these four studies. Each succeeds in finding a cointegrating equation. However, the studies arrive at contradictory results. Consider each of the variables across the four studies.

The first row considers the Balassa-Samuelson (B-S) effect, represented by variable R(NT) the ratio of non-traded/traded goods in the two areas. This is generally measured as the relative CPI/WPI.

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[13] For a discussion of these issues for example, see MacDonald (1999) and (2000).

From equation (4), (4a),(4b), the Balasssa-Samuelson hypothesis is that the real exchange rate R(t)= R(CPI) based upon broad based price indexes such as the CPI is the product of the constant "external" price ratio R(T) of traded goods in the two countries and an "internal" price ratio R(NT). . The "law of one price" for traded goods is that R(T) = C a constant. The ratio R(NT) of nontraded/traded goods in the two countries is called the "internal" price ratio. The weight of non-traded goods in the CPI is fraction w. The B-S hypothesis is that variations in the real exchange rate R(t) derive from variations inn R(NT). That is R(T) is proportional to R(NT).

$$R(CPI) = N(t)p(t)/p^*(t) = R(T)R(NT) \quad (4)$$

$$R(T) = [N(t)p(T;t)/p^*(T;t)] \quad (4a)$$

p(T;t) = price of traded (T) goods at time t.

$$R(NT) = [p(N;t)/p(T;t)]^w / [p^*(N;t)/p^*(T;t)]^w \quad (4b)$$

p(N;t) = price of non-traded (N) goods at time t.

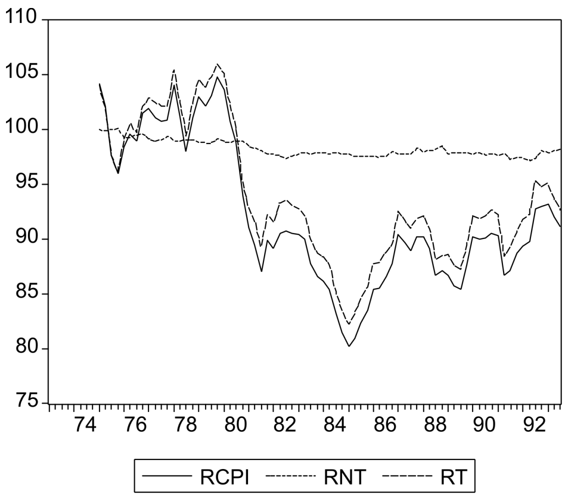
Row 1 in table 1 presents the results of the studies concerning the Balassa-Samuelson R(NT) effect. The [ECB:M] study, column 1, found that the R(NT) effect was statistically insignificant. The study by Clostermann and Friedmann [C-F:1998] in column 3 arrived at a similar result for the DM. Figure 2, derived from [C-F] is a convincing demonstration that the Balassa - Samuelson effect R(NT) has a trivial effect upon the real exchange rate. The curve R(CPI) is the real exchange rate of the DM based upon the CPI. The curve R(T) is the ratio of the prices of traded goods. The curve R(NT) is the Balassa-Samuelson variable. Figure 2 shows that the real exchange rate R(CPI) for the DM is almost identical to the ratio R(T) of traded goods. Both experienced significant variations. By contrast, the internal price ratio R(NT) was practically constant[14]. Duval (2001:346) presents a similar graph

[14] Clostermann and Friedman (1998:213-214) write: "[The figure] shows Germany's relative internal price ratio compared with a trade-weighted average of this group of 10 countries...It is remarkably constant, and - accordingly - the real effective exchange rate on the basis of the overall CPI...seems to be nearly idenical with the real exchange rate based upon prices forv tradables...On bal-

for the Euro. The curve describing the real exchange rate based upon broad based indexes R(t) is almost identical to the external price ratio R(T); hence the internal price ratio R(NT) has a trivial effect upon the real exchange rate.

The papers by Clostermann-Schnatz for the Euro (column 2), and Clark-MacDonald [C-M] for the DM (column 4) arrive at a different conclusion than do [ECB:M] and [C-F] concerning the Balassa-Samuelson R(NT) effect in row 1. Clostermann and Schnatz find that the relative CPI/WPI measure of R(NT) appreciates the synthetic euro, and that the real value of the synthetic Euro and DM were practically identical. On the other hand, Clostermann and Friedmann found that the Balassa-Samuelson R(NT) effect was trivial for the DM. Clark and MacDonald, unlike [C-F], find that the R(NT) effect was significant for the DM.

Figure 2. Alternative measures of the real exchange rate.
$R(CPI) = N(t)p(t)/p^*(t) = R(T)R(NT)$
$R(T) = [N(t)p(T;t)/p^*(T;t)]$ $p(T;t)$ = price of traded (T) goods at time t.
$R(NT) = [p(N;t)/p(T;t)]^w / [p^*(N;t)/p^*(T;t)]^w$



ance so far, not much evidence in favour of a "Balassa-Samuelson effect" in broadly defined real effective D-Mark exchange rates seems to exist in the data under consideration".

Figure 2
Germany, R(CPI)=R(T)R(NT)

How should the contradictions in row 1 be reconciled? One matter is whether variable R(NT) has a statistically significant t-value in a regression with other variables. Another more important matter is whether variable R(NT) is important in explaining variation in R(CPI). The graphs (figure 2) relating the real exchange rate R(CPI) to R(NT) and R(T), presented by Clostermann-Friedman for the DM, and by Duval for the euro are compelling. They show the unimportance of R(NT). It would have been useful if the studies by [C-S] and [C-M] presented similar graphs. One would expect that all would obtain similar graphs.

The second variable is relative productivity in row 2. Column 1 contains the results of [ECB:M]. Since the Balasssa-Samuelson proxy performed poorly as a determinant of long-run exchange rate movement during estimation, as seen in row 1 column 1, the [ECB:M] considered the labour productivity differential between home and abroad. Following Clostermann and Friedmann (1998) labour productivity is defined as the ratio of the real GDP to total employment [y(t) - y*(t)]. The [ECB:M] found that relative productivity is significant and appreciates the real value of the Euro. This result is consistent with that obtained by Clostermann-Friedmann (column 3), and Clark-MacDonald (column 4) for the DM. Relative productivity appreciates the real exchange rate, in the three studies summarized in row 2 columns (1)(3)(4).

By contrast, Clostermann-Schnatz column 2 did not find relative productivity to be significant. They are disturbed by the difference between their study of the synthetic euro and the study by Clostermann-Friedmann for the DM. [C-S: p.9] write: "...the evidence of a more direct productivity variable - approximated, for instance, by the ratio of GDP to the number of employed persons - has also been examined. Although this variable was found to be important in the estimates by Clostermann and Friedmann (1998) for the D-Mark, it has been consistently insignificant in the present estimates for the euro area."

The third and fourth lines concern import prices and/or the terms of trade. Again there are different results in the various studies. [C-S, col. 2] find that the real price of oil depreciates the real exchange rate of the euro. However Clark and MacDonald (col. 4) did not find that the terms of trade affect the real value of the D-Mark.

Only the [C-S] study considered the role of fiscal policy, the ratio of government expenditures/GDP in Europe/US. They found that a rise in fiscal policy depreciates the real value of the currency. This empirical result is quite contrary to the implications of the Mundell - Fleming model. The BEER approach does not aim to explain this apparent contradiction. However, the papers that take the NATREX approach, discussed later, resolve this apparent contradiction.

Net foreign assets, the negative of the net foreign debt, are considered in three of the studies. This variable features in many models of the exchange rate, where a rise in net foreign assets is expected to appreciate the real exchange rate. For example in equation (1) a rise in net foreign assets increases the current account, which tends to appreciate the exchange rate. [ECB:M] find that net foreign assets are not a significant economic variable for the real value of the synthetic euro. This is confirmed by [C-F] who do not find net foreign assets to be significant for the real value of the D-Mark. However Clark-MacDonald obtain a contradictory result. Net foreign assets appreciate the real value of the D-Mark.

The last variable is the real long-term interest rate differential [r(t) - r*(t)]. The results are contradictory. The [ECB:M, col. 1] study of the real value of the synthetic euro found that the long-term real interest rate differential is non-stationary and was included in vector Z(t). The authors are puzzled by the non-stationarity and write: "...the significance of the interest rate differential as a long-term determinant of exchange rate movements seems a bit at odds with economic theory which asserts that real interest rates tend to equalize across countries in the long run. Consequently, the real interest rate differential should not be construed as a long run determinant of exchange rate movements..." Nevertheless, the authors use the interest rate differential to account for medium to longer term movements in the real exchange rate. The study by [C-F, col. 3] of the real value of the D-Mark arrived at the same conclusion. They conclude that a rise in the real interest rate differential significantly appreciates the long run real exchange rate.

On the other, the study of the real value of the synthetic euro by [C-S, col. 2] reached a different conclusion. The real long-term interest rate is a stationary variable. It does not affect the long run real exchange rate, but affects the real exchange rate only in the short run.

How can we resolve the question: is the real long-term interest rate differential stationary/mean reverting or not? In their study of the real value of the D-Mark, Stein and Sauernheimer show (1997:pp. 18-19) that the real long term differential between the German and US real interest rates differs in the periods before and after 1980. After 1980, the differential is stationary and the two real long-term interest rates converge. Prior to 1980, there is not a convergence. Hence the sample period used is important. Using a sample period starting with 1980, the real longterm interest rate differential is I(0), and is only a determinant of the short-term, but not the longterm equilibrium, real exchange rate.

What can we conclude from these five studies? These negative results are confusing. The four BEER studies in table 1 yielded different and often contradictory results, even though each obtained a cointegrating equation with significant values for different vectors of "fundamentals" Z(t). The variables in the cointegrating equations are mixtures of endogenous, control and exogenous variables. Without an explicit theoretical structure it is difficult to know how to interpret the econometric results for the formulation of ECB policy discussed in part 1.

Table 1: *Comparison of BEER Studies*

Real Fundamentals Z(t)	[ECB:M, 2000] Euro	Clostermann-Schnatz (2000): Euro	Clostermann-Friedmann [1998]: DM	Clark-Mac-Donald (1999): DM
R(NT) = Relative (CPI/PPI)	Insignificant	appreciate	Insignificant	appreciate
(y – y*), Relative productivity	appreciate	Insignificant	Appreciate	appreciate
Real price oil	…	depreciate		
Terms of trade				Insignificant
Relative fiscal**	…	Depreciate		
Net foreign assets	Insignificant		insignificant	appreciate
Relative real LT interest, I(1)	Appreciate		appreciate	
SHORTERTERM				
Relative real LT interest, I(0)		Appreciate		

3. Structural Equations determining the Equilibrium Real Exchange Rate: NATREX

In view of the unpromising results above, the [ECB:D] authors went further than the BEER approach, and proceeded to look for structural equations within a coherent theoretical model.

"A further step towards increasing the structure underlying the estimated model is to estimate a number of behavioural relations as commonly found in standard structural macroeconometric models. To begin with, we consider a small-scale model based upon the NATREX approach (NATural Real Exchange rate)....This approach tries to link the real exchange rate to a set of fundamental variables explaining savings, investment and the current account. Natrex is based upon a rigorous stock-flow interaction in a macroeconomic growth [model]. A distinction is made between a medium run equilibrium where external and internal equilibrium prevails (equivalent to the macroeconomic balance approach) and the long-run equilibrium where the budget constraint on net foreign debt is met and the capital stock has reached its steady state level".

[ECB:D] described the NATREX model and estimated several key structural equations. From these equations, they inferred the equilibrium real exchange rate and compared the inferred equilibrium rate with the actual synthetic real Euro. Part 3.1 very tersely describes the crucial structural equations of the NATREX model and the implications for econometric testing. Part 3.2 explains the transmission mechanism linking the endogenous real equilibrium exchange rate to the exogenous and control variables. This is the structure that is ignored in the empirical/econometric studies above. Part 3.3 compares the econometric results of [ECB:D] with the analysis in parts 3.1 and 3.2. Part 4 compares the papers by Duval, Crouhy-Veyrac, Maurin and by Verrue who also use NATREX. The results are summarized in table 2. The two set of studies focus upon different aspects of the model. Whereas the set summarized in table 2 estimate a dynamic reduced form equation for the real exchange rate, the [ECB:D] estimates structural equations but not the reduced form equation for the exchange rate[15]. The two sets of studies based upon the NATREX are mutually consistent.

3.1. The Crucial Equations of the NATREX model[16]

The NATREX is the equilibrium real exchange rate as defined in part 1 above. The NATREX is not a point, but is a trajectory associated with both internal and external balance. Equation (2) for macroeconomic balance, or internal equilibrium, is equation (5): Saving less investment is equal to the current account, evaluated at

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[15] Verrue(1998) estimated both structural equations as well as the reduced form equation for the Belgian franc. Gandolfo and Felettigh estimate a system of simultaneous nonlinear dynamic equations using the FIML technique for the Italian Lira. They write that: "Our estimates confirm the validity of the NATREX theory for the Italian economy. In particular, our in-sample simulations for the long-run equilibrium real exchange rate confirm the analysis of the real misalignment of the lira made by the Bank of Italy."

[16] The reader is directed to the following references for a full exposition: Stein, Allen et al (1997 ed), Stein (1994), Stein (1999), Stein and Paladino (1999). I use Stein (1999) as the latest thinking on the subject.

capacity output. Except for the exchange rate and real long-term interest rate, measure each variable as a ratio to capacity output. So ial (private plus public) consumption c(t) depends positively upon net worth equal to capital less net foreign debt F(t), and upon fiscal policy which is government consumption g(t), and the vector of tax rates $\tau(t)$. Social saving s = 1 - c depends positively upon net foreign debt (F), and upon fiscal policy (g,τ). Write saving as $s = 1 - c = S(c(g,\tau), F)$. The positive relation between social saving, by the sum of firms, households and government, and net foreign debt is a stability condition for "intertemporal optimization".

Investment depends basically upon the Keynes-Tobin q-ratio: the present value of expected profits, divided by the supply price of investment goods. The q-ratio depends upon foreign demand and the marginal costs of production. The view taken here is that the firms sell in a world market, where the negatively sloped demand curve is exogenous to the country. Foreign demand is reflected in foreign[17] social consumption c* and by the terms of trade T. Marginal costs of production depends positively upon the real exchange rate R(t), and negatively upon the level of productivity y(t). The real exchange rate R(t) negatively affects expected profits because a rise in R raises domestic prices and costs[18] relative to world demand. Marginal costs rise, profits decline, the q-ratio is reduced and investment is discouraged. Investment is $I(t) = I(R(t)\ y(t),T(t),c^*(t),r(t))$, where r is the real rate of interest.

The current account CA is the trade balance B(t) less net "interest payments" r(t)F(t), where F(t) is net foreign "debt", or net liabilities to foreigners in the form of debt plus equity. The "interest rate" r(t) should also represent the dividend rate, so that r(t)F(t) is net income transferred abroad. The trade balance is negatively related to the real exchange rate for the usual reasons. Productivity y

[17] Foreign variables are denoted by an asterisk.

[18] Stein (1999) measures the real exchange rate $R(t)=N(t)w(t)/w^*(t)$, where w is unit labor costs. Then the exchange rate appreciation clearly raises marginal costs and discourages investment.

(t) increases the trade balance because it lowers the marginal cost and increases the supply curve of tradable. Marginal cost is equal to world demand at a larger output of tradable. Foreign demand c* generates world demand for the exports of the Euro area. The current account function is CA=C(R,c, y,F,r;c*), where the derivatives of c and c* reflect the marginal propensity to import associated with a rise in the consumption ratio. Internal balance at capacity output (u = 0) is equation (5).

$$S(c(t),F(t)) - I(R(t),y(t),r(t),T(t)) = CA(R(t),c(t),y(t),F(t),r(t);c^*(t)) \mid u = 0. \quad (5)$$

Portfolio balance at the longer run equilibrium real exchange rate implies that domestic and foreign real long-term interest rates are equal, or differ by a constant. This is one external equilibrium condition.

$$r = r^*. \quad (6)$$

Solving (5) and (6) for the medium run equilibrium exchange rate implies equation (7). This is the same equation that is used in the macroeconomic balance approach.

The NATREX model is a generalization of the macroeconomic balance model, insofar that we link the medium run and the trajectory to the longer run equilibrium. Exogenous and control variables Z(t) = [c(t),c*(t),y(t),T(t),r*(t)] have different effects in the longer run than they do in the medium run macroeconomic balance equation. In particular, *the Mundell-Fleming analysis of the effects of fiscal policy is only valid in the medium run, and the conclusions are reversed in sign for the longer run.*

The linkage of the medium to the longer run arises from two dynamic equations concerning the net foreign debt plus net equity, which we call "debt", F(t) and the level of productivity y(t), equations (8) and (9). These two equations, when added to the equation (7), represent the NATREX model. The current account CA(t) is the rate of change of the net claims on foreigners in the form of foreign debt plus equity, equation (8). The growth of productivity dy(t)/dt is directly related to the rate of investment, equation (9).

BASIC EQUATIONS OF THE NATREX MODEL

$$R(t) = R(c(t), c^*(t), y(t), F(t), r^*(t), T(t)). \quad (7)$$

$$dF(t)/dt = I(R(t),y(t),r^*(t), T(t)) - S(F(t),y(t);c(t)) =$$

$$= - CA(R(t),y,(t),F(t),r(t);c(t),c^*(t)) \quad (8)$$

$$dy(t)/dt = V[I(R(t),y(t),r^*(t)),T(t))], V' > 0. \quad (9)$$

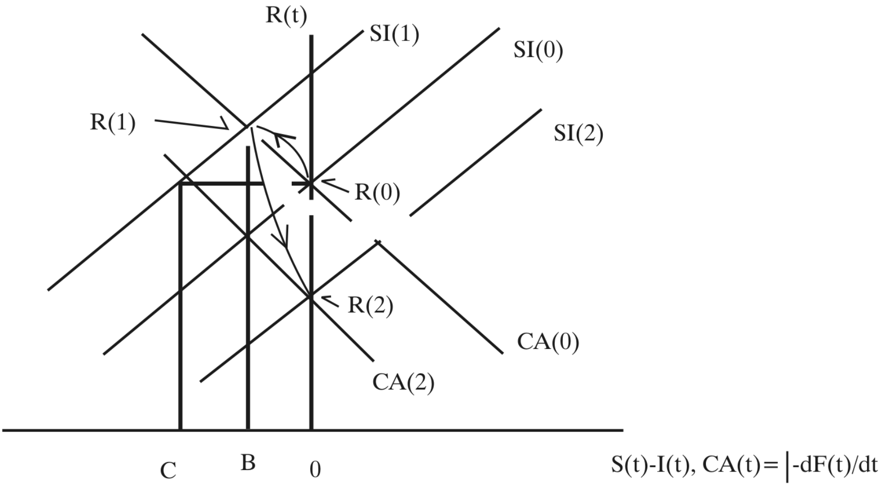
(–F(t)) = net claims on foreigners in the form of debt plus equity; R(t) = real exchange rate; y(t)= productivity; c(t) = social consumption/GDP; T(t) = terms of trade; r*(t) = real long-term rate of interest (dividend rate).

Equations (7),(8) and (9) constitute the core of the NATREX model. The exogenous variables: foreign "time preference c*(t), foreign real long term rate of interest r*(t), terms of trade T(t). The control variables are: c(t) domestic social consumption (often referred to as "time preference"). Fiscal policy [g(t), τ(t)] and [g*(t), τ^*(t)] are important determinants of time preference: the ratio of private plus public consumption to GDP. Thrift is 1–c. We write c(t) = c(g(t), τ(t)) and c*(t) = c*(g*(t),τ^*(t)). In the theoretical NATREX model, the endogenous variables are: the real exchange rate R(t), "debt" F(t) and capital or productivity y(t). In the econometric analyses, differential "productivity" [y(t) – y*(t)] is treated as exogenous.

In order to evaluate and interpret the econometric results of all of the papers, summarized in tables 1 and 2, it is necessary to explain the economic and econometric implications of equations (7), (8) and (9). This is the subject of section (3.2).

3.2. Transmission Mechanism: How changes in thrift/time preference/fiscal policy at home and abroad affect the NATREX

Figure 3 is a simple[19] graphic exposition of the transmission mechanism in the model, and permits us to understand the distinction between exogenous, control and endogenous variables. The curve SI relates saving less investment to the real exchange rate, and curve CA relates the current account to the real exchange rate. They are evaluated when real interest rates have converged, equation (6).Their intersection gives us the medium run equilibrium exchange rate, equation (7).



Rise in social consumption c.

Long run: R(0) – R(2), cointegrating eqn.; Medium run R(1) – R(2), correction; impact R(0) – R(1).

The negatively sloped CA curve describes how an appreciation of the real exchange rate decreases the trade balance and current account. The positively sloped SI curve describes how an appreciation of the real exchange rate raises domestic costs and prices relative to world demand and reduces the present value of expected profits. The Keynes-Tobin q-ratio declines, and investment then

[19] See Stein (1997, ch.2) for the full analysis where both F(t) and capital k(t) or productivity y(t), are both endogenous dynamic variables.

declines relative to saving. That is why S-I rises with the real exchange rate.

Initially, saving less investment is described by curve SI(0), and the current account by curve CA(0). Real exchange rate R(0) produces internal balance, when there is portfolio balance $r(t) = r^*(t)$.

The difference between the Mundell-Fleming (M-F) macroeconomic balance approach and the NATREX model is seen clearly by considering the effects of an expansionary fiscal policy upon the real exchange rate. Whereas the M-F model claims that the real exchange rate will appreciate, the NATREX model claims that the medium run appreciation will be more than offset in the longer run. An expansionary fiscal policy will depreciate the longer run exchange rate.

Let control variable fiscal policy/time preference c(t) rise. Social consumption rises, and social saving declines relative to investment. The SI curve shifts from SI(0) to SI(1). If all of the increased demand were directed to home goods, then the CA curve is unaffected[20]. At exchange rate R(0), the ex-ante current account CA(0) now exceeds ex-ante saving less investment SI(1) by 0C. Investment less saving is the desired capital inflow.

Several things happen. The excess demand raises the domestic interest rates. Domestic firms/government borrow abroad what they cannot borrow at home. The desired capital inflow 0C tends to appreciate the exchange rate to $R(1) > R(0)$, to restore internal and external balance. This is a movement to the medium run equilibrium, evaluated at the given level of net foreign assets F(t) and productivity y(t). So far, this is the same as the M-F macroeconomic balance approach.

The transition to the longer run resulting from a decrease in social saving by quantity 0C results from two crowding out effects[21].

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[20] The marginal propensity to import is a small fraction. This means that the CA curve shifts to the left by less than the SI curve.

[21] The two crowding out effects are stressed in Stein (1999), but only one was discussed in the earlier NATREX papers.

The first is that the appreciated exchange rate R(1) is associated with a current account deficit 0B. The foreign debt F(t) rises to F(t) + (0B)dt, equation (8). The second crowding out effect results from the effect of the appreciated exchange rate R(1) upon the rate of investment. The latter is crowded out by (CB)dt. The decline in the rate of investment adversely affects the growth of productivity, equation (9). The adverse effect is: $\Delta[dy(t)] = V'I_R \Delta R < 0$.

The growth of the foreign debt increases interest payments and depresses the CA curve towards CA(2). At the existing real exchange rate, the current account declines. The steady decline in the CA function, arising from the growing debt F(t) depreciates the exchange rate. The real exchange rate depreciates steadily as the CA curve declines along the SI curve.

The decline in the growth of productivity, resulting from the crowding out of investment, adversely affects the current account function. Given the growth of the real wage, a general decline in the growth of productivity raises marginal costs in all of the sectors. Given the growth in world demand, the rise in marginal costs, due to the decline in the growth in productivity, shifts the current account function to the left.

Combining the two crowding out effects, an expansionary fiscal policy leading to a rise in social consumption c(t) shifts the CA function from CA(0) to the left and down, in figure 3. If the SI function did not shift to the right, the exchange rate would continue to depreciate and the foreign debt would explode. A necessary condition for intertemporal stability is that the rise in the foreign debt decreases net worth, which decreases social consumption and increases social saving, by the sum of firms, households and government. In a dynamically stable system the growth of the debt must lead to a rise in saving which shifts the SI curve to the right in the direction of SI(2). The longer run equilibrium occurs where saving less investment SI(2) intersects current account CA(2). There is a higher steady state debt and lower level of relative productivity in the new longer run equilibrium. The series of medium run equilibria produces trajectory R(0)–R(1)–R(2) of the exchange rate to the longer run equilibrium.

The effect of an "exogenous" rise in productivity is more complex and ambiguous. It has been discussed elsewhere[22] in detail. Initially, investment rises relative to saving: the SI curves shifts to the left. The real exchange rate appreciates and *the capital inflow* or *trade deficit finances investment*. Eventually, the economy is more productive/competitive and the current account function shifts to the right. Insofar as the higher level of productivity eventually shifts the current account CA function to the right by more than it shifts the saving less investment, the long run real exchange rate appreciates.

3.3. [ECB:D] Empirical Results[23] based upon structural equations

The ECB:D estimated VEC models for the variables entering the investment, consumption and trade balance equations. The object was to examine the structural equations (10)–(12) and from them estimate the equilibrium real exchange rate.

Their consumption equation is (10), and the implied saving function is S(t)/Q(t) = 1 – C(t)/Q(t). The ratio of consumption to output C(t)/Q(t) depends positively upon net worth; capital less debt. Hence C(t)/Q(t) depends negatively upon the foreign debt/output F(t)/Q(t). Insofar as the current account deficits are financed through debt rather than equity, the effect of cumulative current account deficits is built in. The stability of the system de-

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[22] Stein, Allen et al, pp.24–26, table 2.1 p.46, pp.66–67 table 2.3; MacDonald and Stein, p.16.

[23] I am using the authors' notation, except for the growth in total factor productivity. I am not specifying when they use the long-term or the short-term interest rates.

pends crucially upon the sign of the debt variable: social saving (consumption) must rise (fall) with the debt. The authors also assume that C(t)/Q(t) depends positively upon the growth of total factor productivity[24] a*, negatively upon the real rate of interest r(t), and positively upon the nominal interest rate i(t), which represents the business cycle.

Their investment function equation (11) reflects a declining marginal product of capital and an estimate of the q-ratio. Investment/output I(t)/Q(t) is negatively related to the capital stock/output K(t)/Q(t) and to the real rate of interest r(t), and is positively related to a*, the growth of total factor productivity. Investment is negatively related to the real exchange rate. This is the investment crowding out effect, which produces a positively sloped SI curve in figure 3 above.

The trade balance equation (12) states that the trade balance TB(t)/Q(t) is negatively related to the real exchange rate R(t), domestic social consumption ratio C(t)/Q(t) given in (10), and positively related to foreign social consumption ratio C*(t)/Q*(t) and to the growth of total factor productivity. Three equations are used, but not estimated directly. One is the uncovered real interest parity equation. In addition, there are two dynamic equations. One is the growth of the debt/GDP ratio F(t)/Q(t) and the second is the growth of capital/GDP ratio K(t)/Q(t).

[ECB: D] Structural Equations Estimated

$$C(t)/Q(t) = 1 - S(t)/Q(t) = a_6 + a_7 a^* - a_8 F(t-2)/Q(t-2) - a_9 r(t) + a_{10} i(t-2) \quad (10)$$

$$I(t)/Q(t) = a_1 + a_2 a^* - a_3 K(t)/Q(t) - a_4 r(t) - a_3 R(t-3) \quad (11)$$

[24] The production function is: $Y(t) = A(t)K(t)_{\alpha}L(t)_{\beta}$. The growth of total factor productivity is
$a^* = [dA(t)/dt]/A(t)$.

$$TB(t)/Q(t) = a_{11} - a_{12}R(t) - a_{13}\, a^*(t) - a_{14}\, i(t) + a_{15}\, F(t)/Q(t)$$
$$+ a_{16}\, r(t-2) + a_{17}\, C^*(t-4)/Q^*(t-4) \quad (12)$$

Hypothesized values of regression coefficients are positive. C/Q = social consumption/GDP, I/Q = gross social investment/GDP, TB/Q = trade balance/GDP.

The [ECB-2] authors estimated separate VEC models for the variables involved in equations (10)-(12) over the period 1972:1 - 1997:4. The empirical results were as follows. The variables are of order I(1) except for the productivity growth rate. There is one cointegrating equation for each behavioral equation. There are certain crucial requirements for the validity of the structural aspects of the NATREX model, and others are not crucial. All of the crucial coefficients have the hypothesized sign and are significant. Results (a) - (d) below show that the crucial structural equations are consistent with the evidence.

(a) The rate of investment is negatively related to the real exchange rate. Exchange rate appreciation crowds out domestic capital formation in the estimate of equation (11). This is consistent with the positively sloped SI curve in figure 3.

(b) The trade balance is negatively related to the real exchange rate in the estimate of trade balance equation (12). Exchange rate appreciation crowds out the trade balance and tends to raise the debt. This is consistent with the negatively sloped CA function in figure 3.

(c) The stability of the system requires that the foreign debt reduce wealth, which reduces social consumption by the sum of households, firms and government. The debt significantly reduces social consumption in the estimate of social consumption equation (10). This is consistent with the dynamics R(1)-R(2) in figure 3.

(d) A rise in the capital/output ratio reduces the rate of capital formation, in the estimate of investment equation (11).

The medium run equilibrium real exchange rate can be derived from a solution of S(t) – I(t) = CA(t), using the estimates from (10)–(12) above, evaluated with the current debt F(t)/Q(t) and capital K(t)/Q(t). The [ECB:D] derives the longer run equilibrium real exchange rate by adding the conditions that: the current account deficit/debt be a constant, and that the ratio of investment to capital be a constant. The model is then simulated to compare the actual with the simulated estimates outside the sample period. The [ECB:D] simulation results and conclusions are as follows.

"Overall, the variability of the estimated equilibrium rates is surprisingly high. On the positive side, this could be used to refute the claim that exchange rate models based upon fundamentals are always at a loss in explaining actual changes because fundamentals are not volatile enough. The equilibrium estimates at the end of 1999 of our four NATREX simulations diverge by +/– 4%. Still, the basic pattern of the synthetic euro exchange rate has nevertheless been traced. It remains to be seen if the increasing undervaluation since 1997 for the medium run equilibria is due to an out-of-sample breakdown of the model."

There are several questions that should be posed concerning the method of solving the estimated structural equations to derive an estimated equilibrium exchange rate. First and foremost is the correspondence between theoretical and empirical variables. In the model, the debt F(t) is an integral of current account deficits, adjusting for the interest rates. Some of the deficit will be financed by debt and some by equity. Hence F(t) is not the gross foreign debt, but is net claims on foreigners in any form. Second, capital is very difficult to measure. The authors use measures of depreciation, but obsolescence is much more important, as any owner of a computer knows. Vintage models of capital attempt to circumvent this problem. In the vintage models, capital is not the integral of investment adjusted for depreciation. Third, there is a range for estimates of the coefficients, depending upon the method of estimation used. This point will be stressed in our discussion underlying the estimates from reduced form models. Consequently, the solution of the estimated model, involves the multiplication of lots of uncertain estimates. That is, the process of inverting the matrix tends to produce great instabilities of results. Fourth, there are some puzzling results. For example, the rate of growth of total factor productivity is not significant in the estimated investment equation. My conclusion is that [ECB:D] has shown that: (i) the crucial transmission mechanisms of the NATREX model are consistent with the evidence, but (ii) one should be hesitant in accepting the quantitative results from the simulation as precise estimates.

4. Reduced Form Dynamic Equation for NATREX

4.1. The relation between VEC econometrics and NATREX theory

The studies by Duval, Crouhy-Veyrac, Maurin and Verrue use the NATREX model to obtain estimates of the reduced form dynamic equation for the equilibrium real exchange rate. The NATREX model is a stock-flow dynamic model, where a distinction is made between the medium term and the longer- term trajectory of the exchange rate. In the medium term, the stock of debt and level of productivity are given, but they are endogenous in the longer run. A clear distinction is made between endogenous variables, exogenous variables and control variables, in establishing the trajectory of the exchange rate. The BEER approach does not do this. Finally, the NATREX approach is easily related to the VEC techniques in econometrics.

Equation (3) for the "equilibrium" exchange rate R(t) can be generalized to equation (13). The term BZ(t) is the longer run equilibrium associated with the "fundamentals" Z(t). Insofar as R(t) and vector Z(t) are integrated I(1), BZ(t) is the cointegrating equation. The second term a[R(t-1) - BZ(t-1)] is the error correction (EC) term. The third term represents short-term shocks from variables that are stationary, transitory variables I(0) and have zero expectations.

$$R(t) = BZ(t) + a[R(t-1) - BZ(t-1)] + \Sigma b(i)\Delta Z(t-i) + \varepsilon(t) \qquad (13)$$

In the NATREX model graphed in figure 2, the cointegrating equation
R(t) = BZ(t) reflects the long run movement from R(0) to R(2), resulting from a rise in time preference. The EC term represents the movement from R(1) to R(2), resulting from endogenous variations in stocks. The third term represents the movement from R(0) to R(1).

There are several ways that vector B has been estimated. One way is to use a NLS method due to Phillips-Loretan to estimate B directly from (13). Another way is directly with the Johansen/VEC method equation (13a), which can be derived from (13). The tests involve the following questions. (a) Are R(t) and vector Z(t) integrated I(1)? (b) Is there just 1 cointegrating equation? (c) Are the Z's weakly exogenous?

$$\Delta R(t) = \alpha[R(t-1) - BZ(t-1)] + \Sigma b(i)\Delta Z(t-i) + \varepsilon(t) \qquad (13a)$$

A third method is the Engle-Granger 2-stage least squares. After establishing that [R(t),Z(t)] are I(1) and cointegrated, an OLS estimate of B is done directly. Then the residual [R(t-1) - BZ(t-1)] is used as the second term in (13).

4.2. Measurement of the Variables [25]

A difficult problem, handled differently by the various authors, is how to measure the variables. Figure 4 displays the basic I(1) variables, primarily as four quarter moving averages (MA). These variables are the real exchange rate R(t), relative prices p(t)/p*(t) and the disturbances to productivity and thrift that produce the change in the real exchange rate from R(0) to R(2).

[25] I am using the data provided by Liliane Crouhy-Veyrac.

Figure 4. Variables that are I(1), that do not revert to a constant mean. MA = 4Q moving average. R = real exchange rate = Np/p* = EUUSREDPMA;
N = nominal exchange rate $US/synthetic Euro = EUUSNER-MA; c/c* = Euro/US ratio social consumption/GDP = EU-USCRATMA; y/y* = Euro/US labor productivity = GDP/employment; p/p* =
= Euro/US GDP deflators = EUUSPDMA;
g/g* = Euro/US government consumption/GDP = EUUSGOV-RATMA

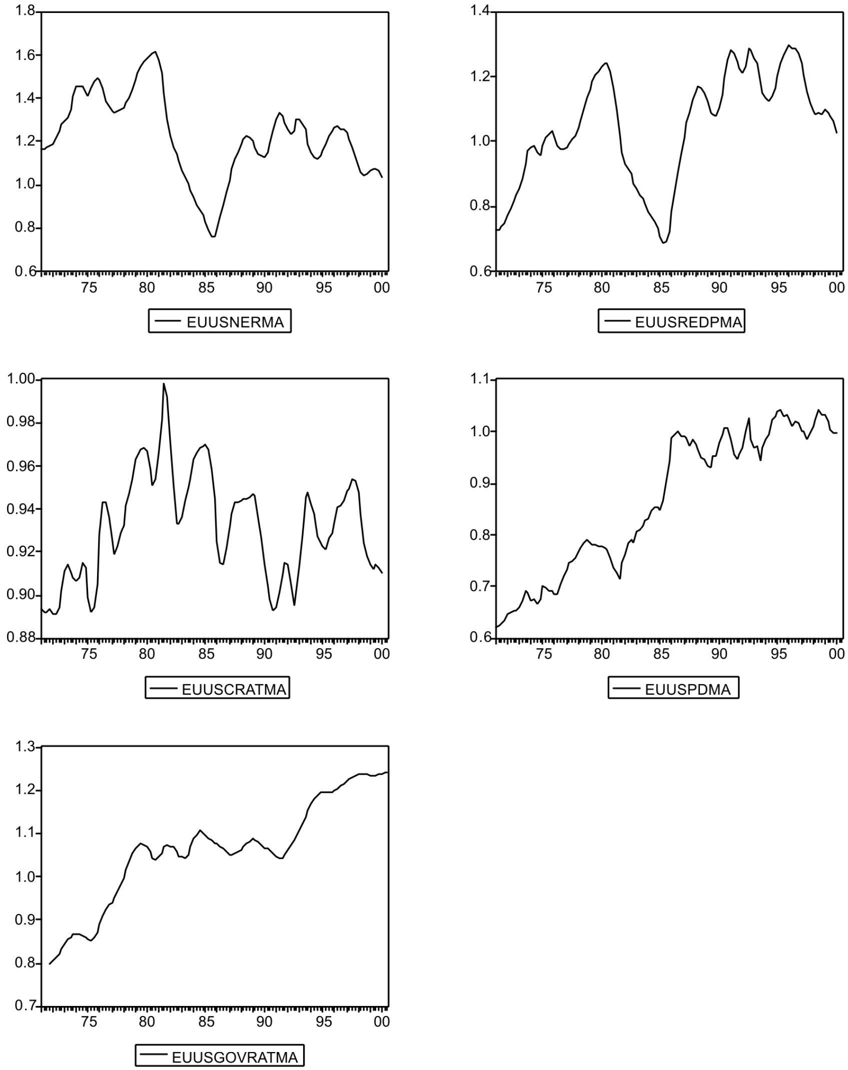

Figure 4
Variables I(1)

(a) The real exchange rate R(t) can be measured in terms of GDP deflators or relative wages, above. Since 1973 the two measures are very similar for the synthetic Euro-$US, but they are quite different earlier.

(b) There are alternative measures of social "time preference". Theoretically, the measure should be the ratio c(t) of private plus public consumption to GDP. A question is raised whether this variable is truly exogenous. We wrote that c(t) = c(g(t), τ(t)), where g(t) is government consumption/GDP and τ(t) is a vector of tax rates. Some authors useg(t) as their measure of time preference, since it is more exogenous than c(t). Figure 4 shows that the ratio EUUSCRATMA = c(t)/c*(t) of EU to US social consumption/GDP is quite different from the ratio EUUSGOVRATMA = g(t)/g*(t) of government consumption/GDP. The ratio g(t)/g*(t) misses the effects of changes in tax policy upon social consumption.

(c) The measurement of the productivity disturbance is difficult. Theoretically, we want to find a measure for a factor that: initially raises the productivity of capital, induces investment that eventually raises output and lowers the marginal costs of tradable goods. We seek a relatively exogenous factor that ultimately shifts the CA function to the right by more than it shifts the SI curve in figure 3. Some authors use the productivity of labor y(t) = GDP/employment. Others use total factor productivity q(t), the Solow residual. It is theoretically appealing to use the differential rate of return on investment, but this is a stationary variable both in Europe and in the US. In part 5 below, we consider this variable among the I(0) disturbances that produce the change from R(0) to R(1).

(d) The NATREX model per se ignores the shorter-term transitory I(0) effects that converge to zero. The shorter run disturbances involve changes in the fundamentals and the levels of disequilibrium terms contained in the vector ΔZ(t) mentioned above. The net effect of changes in the disequilibrium terms ΔZ(t) is to change: interest rates, the rate of capacity utilization or the return on investment[26].

[26] See the discussion in Stein (1999) relating the Keynesian approach with the NATREX.

Different studies use different variables for the I(0) effects, graphed in figure 5. Some use the real long-term interest rate differential [r(t) – r*(t)], as in equation (3) above. It is generally found that this differential is stationary and converges to zero. Some use the nominal long-term interest rate differential [i(t) – i*(t)], as a control variable. We use the differential rate of return on investment b(t)– b*(t) = EUUSRETURN as an important I(0) variable. Another candidate for a disequilibrium variable is the deviation u(t) of the rate of capacity utilization from its longer term stationary mean.

Figure 5. Stationary, mean reverting I(0) variables. EUUSRETURN = b(t) - b*(t) = growth rate/investment ratio in Europe less US; EUUSOUTGAP = u(t)-u*t) = output gap, actual/potential GDP, in Europe less US; EUUSLINT = long term nominal interest rate in Europe less US = i(t) - i*(t); EUUSGROW = Europe less US growth rate of GDP.

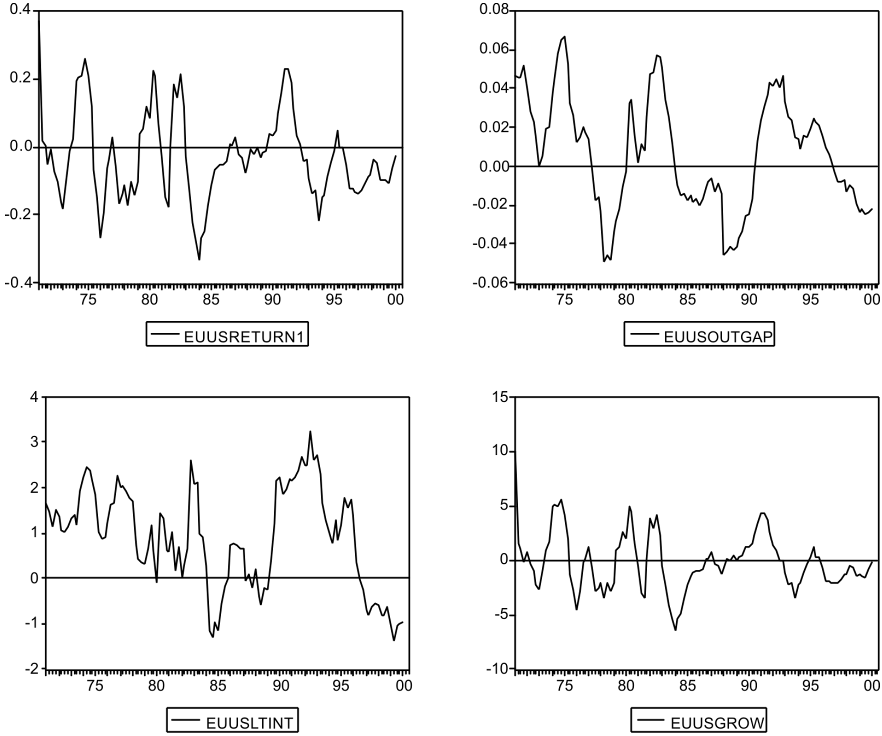

Figure 5
Variables I(0)

4.3. Summary of the Studies for the Real Exchange Rate

Table 2 columns (1)-(4) summarize in a comparable way the results of the four studies cited above. The explanatory power of the model for the nominal rate is discussed in section 5. In all of the studies, the relations among variables R(t) and Z(t) pass the econometric tests mentioned above. The qualitative significant sign results are similar, but the values differ according to the econometric method used.

All find that the ratio c(t)/c*(t) of Euro/US social consumption/GDP depreciates the longer run value of the Euro. This corresponds to the movement from R(0) to R(2) in figure 2. Crouhy-Veyrac and Maurin considered the ratio g(t)/g*(t) of government consumption/GDP as an exogenous component of social consumption.[27] It is always true that a rise in g(t), the European government consumption/GDP, depreciates the longer run value of the Euro. In the Crouhy-Veyrac study, a rise in US government consumption/GDP g*(t) appreciates the euro significantly when the Engle-Granger method is used, but not when the VEC method is used.

In the Crouhy-Veyrac study (column 2), the relative productivity variable measured as relative GDP/employee = y(t)/y*(t) in Europe/US appreciates the long run value of the euro, but it is not significant in Verrue's study (column 3) and in one of Maurin's studies. Duval and Maurin use relative Europe/US total factor productivity (Solow residual) denoted q(t)/q*(t). They find that relative total factor productivity appreciates the long run equilibrium value of the Euro.

The lower part of table 3 concerns the transitory and disequilibrium variables: $\Delta Z(t)$ in equation (13). Duval and Maurin find a strong confirmation of the trajectory R(0)-R(1)-R(2) in figure 2, resulting from a rise in social consumption. As mentioned above a rise in c(t)/c*(t), relative Europe/US time preference, depreciates the long run real value of the euro: trajectory R(0) - R(2). However, the medium run effect of Δ(c(t)/c*(t)) appreciates the euro: trajectory R(0) - R(1).

Duval and Maurin also find that the medium run productivity effect Δ(q(t)/q*(t)) appreciates the Euro. The NATREX interpretation is that a rise in relative productivity raises the Keynes-Tobin q-ratio, stimulates investment relative to saving and shifts the SI curve in figure 2 to the left. The real exchange rate appreciates and leads to a trade deficit to finance the excess investment over saving. The longer run effect[28] is that the rise in productivity shifts the CA

[27] Crouhy-Veyrac used many different measures, and I am only citing one part of her results. Maurin used relative government deficits.

[28] See MacDonald and Stein (1999) p. 16, and Stein and Allen (1997), pp. 24-26, p. 67 table 2.3 and the discussion in that section, concerning the effect of a rise in productivity.

function to the right by more than it shifts the SI function, in figure 2 above[29].

The transitory disequilbrium effects are captured by the interest rate differential term in equation (3). Duval and Verrue use the real long-term interest differential [r(t)-r*(t)], which is I(0) and converges to zero. In all cases, a rise in the euro-US real long-term appreciates the euro, This effect is temporary, since all four authors find that the differential converges to zero. Crouhy - Veyrac uses the long-term nominal interest rate differential [i(t) - i*(t)]. There can be several justifications for this. First: the nominal interest rate is more of a control variable than is the real rate. Second, prices change slowly. She finds that the long-term nominal interest rate differential appreciates the euro.

These studies confirm the implication of the NATREX model as opposed to the Mundell - Fleming model. The M-F model claims that an expansionary fiscal policy that leads to a rise in social consumption appreciates the real exchange rate. The NATREX model claims that such a policy appreciates the real exchange rate in the medium run. In the longer run, the real exchange rate depreciates below its initial level.

Table 2: *Comparison/Summary of Reduced Form Dynamic Equations for real R(t) exchange rate of Euro, rise is an appreciation of Euro.*

	(1)	(2)	(3)	(4)
Fundamental variable: Z(t); I(1)	Duval: 1970-1999	Crouhy-Veyrac: 1973:1-2000:2	Verrue: 1977:1-1998:3	Maurin 1975:1-1997:2
Long run real Natrex				

29 Some authors, particularly Duval, make a distinction between the tradable and non-tradable sectors and add the Balassa-Samuelson effect as a determinant of the long-term equilibrium real exchange rate. We showed in figure 2 above that the ratio of non-traded/traded goods prices has negligible explanatory power.

Time preference: Social consumption/GDP c(t)/c*(t); Government consumption/GDP g(t)/g*(t) theory: depreciate		c/c* depreciate	VEC: g depreciates; (g*?) deprec; E-G: g depreciates; g* appreciates	c/c* depreciates	c/c* depreciate (g-g*) depreciate
Productivity; y(t)/y*(t); total productivity q/q* theory: appreciate	labor factor	q/q* appreciate	y/y* appreciate	n.s.	q/q* apprec
Domestic/foreign index p(t)/p*(t) PPP, depreciate	price				
Change dZ(t), Transitory I(0) variables					
d(time preference) = d(c/c*) theory: appreciate		d(c/c*) appreciate			d(c/c*) appreciate d(g-g*) appreciate
d(productivity) d(y/y*);d(q/q*) theory: appreci-	=	d(q/q*) appreciate			d(q/q*) appreciate

ate (domestic – foreign)				
interest rate: nominal (i(t)–i*(t)); real (r–r*) theory: appreciate		(r–r*) appreciate	(i–i*) appreciate	(r–r*) appreciate

5. The Nominal Value of the Euro

The ultimate object of research concerning the Euro is to answer the following questions: (#1) What is the equilibrium trajectory of the nominal euro, measured as dollars/euro? (#2) To what extent has the equilibrium nominal euro been determined by relative prices (PPP), and to what extent has it been determined by the real NATREX? (#3) How important have been the transitory factors in affecting the value of the euro? (#4) Is the euro currently undervalued, and by what criteria? Our answers are the subject of this part.

The nominal value of the euro N(t) is defined by equation (14). It is the real value R(t) times the ratio p*(t)/p(t) of foreign to euro prices. The *medium run NATREX real* exchange rate is $R_m(Z(t),t)$ in equation (15). The first part is the longer run equilibrium BZ(t), a linear combination of the fundamentals Z(t) = [c(t)/c*(t), y(t)/y*(t)] discussed above. The second term, the trajectory, corresponds to the serially correlated error correction (EC) evaluated "h" periods earlier: term [R(t–h) – BZ(t–h)].

RESEARCH DESIGN FOR THE NATREX NOMINAL EXCHANGE RATE

$$N(t) = R(t)[p^*(t)/p(t)] \quad (14)$$

$$R_m(Z(t),t) = BZ(t) + B_1 [R(t-h) - BZ(t-h)] \quad (15)$$

$$\log N^*(t) = \log R_m(Z(t),t) - \log [p(t)/p^*(t)] \quad (16)$$

$$\log N(t) = \log R_m(Z(t),t) - \log [p(t)/p^*(t)] + a_2[b(t) - b^*(t)] + \varepsilon_1(t) \quad (17)$$

N(t) = nominal exchange rate $US/euro; R(t) = real value euro = Np/p^*; Euro/US GDP deflators = $p(t)/p^*(t)$; BZ(t) = longer run real NATREX; $R_m(Z(t),t)$ = medium run real NATREX; $N^*(t)$ = Nominal NATREX; Differential rates of return on investment = $[b(t) - b^*(t)]$.

Combining equations (14)-(15), the nominal NATREX exchange rate $N^*(t)$ can be expressed as equation (16). The first term $R_m(Z(t),t)$ is the real medium run NATREX in (16). The second term $p(t)/p^*(t)$ is relative price term: the PPP. The actual nominal exchange rate N(t) is equation (17), which combines $N^*(t)$ the medium run NATREX with a generalization of the "interest rate parity" theory. The third term reflects the I(0) differential rates of return on investment. For example, they may be the effects of perturbations to productivity and thrift that generate rates of return differentials and produce the movement R(0)-R(1). Based upon the estimates of equation (17), we propose an answer to questions #1 - #4 above.

Outline and conclusions

The following conclusions are derived in the sections below. *First:* In tables 3,4 we estimate the longer-run real NATREX. We use variables that are objectively measurable and easy to calculate. The longer run real NATREX rate depends upon the fundamentals Z(t) of productivity and relative social consumption. The qualitative, significant sign, results are robust to the econometric method used, but the quantitative results are sensitive to the method of estimation. The OLS method (table 4) produces a better fit than does the VEC-Johansen method (table 3).

Second: In table 5 we estimate Rm[Z(t),t] the medium run NATREX in equation (15). This value is determined by the long run NATREX and by the error correction.

Third: In table 6 we estimate the *nominal* exchange rate equation (17). The logarithm of the nominal rate N(t) is equal to the logarithm real medium run NATREX minus the logarithm of relative prices plus the shorter run I(0) disturbances to productivity and thrift. Since these I(0) disturbances are correlated we subsume all of the economically pertinent I(0) perturbations under the differential of rates of return on investment, denoted [b(t) - b*(t)]. Table 6 is an encouraging confirmation of the nominal NATREX theory.

Fourth: There is structural stability. The recursive estimates of the coefficients of the medium run nominal NATREX equation (17), estimated over the entire period 1971:1 - 2000:1, are stable from1985:01-2000:01. The **nominal** exchange rate converges to the medium run nominal NATREX. (iii)The deviation can be called "misalignment". The misalignment of the current euro is less than 1 standard deviation of the value of the error over the entire period.

5.1. The NATREX Value of the Real Euro

The object of this section is to derive the long and medium run values of the *real* NATREX. We show to what extent the econometric results are robust to different measures of the variables and estimation techniques. Our information set to explain N(t) or R(t) consists of objective and easily calculable measures of the real fundamentals, which are known at time t. The real exchange rate R(t) is based upon the GDP deflators, the broadest measure of prices. No distinction is made between traded and non-traded goods, for the reasons cited in the criticism of the Balassa-Samuelson hypothesis. For "time preference" we use the ratio of social consumption/GDP in Europe relative to the US, denoted c(t)/c*(t). The productivity variable is measured directly and simply as GDP to employment y(t)/y*(t), rather than as the ratio of total factor productivity q(t)/q*(t). The latter is an indirect measure based upon the sum of the Solow residuals from a Cobb-Douglas production function of "capital" and labor. The rationale for using total factor productivity, the sum of Solow residuals, is sensible. There are some difficulties. The measure of capital is arbitrary, in view of the intractable problems of accounting for obsolescence and depreciation. Moreover, one never knows what is the correct aggregate production function. For example, a vintage model makes more sense than one with "capital" that is the sum of past investment.

Tables 3 and 4 present two estimates[30] of the longer run *real* NATREX, denoted BZ(t), where Z(t) is the vector of the ratios of productivity and thrift in the two areas. Table 3 is based upon a VEC-Johansen method of estimation. The three variables R(t)= EUUSREDPMA, [c(t)/c*(t)] = EUUSCRATMA and [y(t)/y*(t)] = EUUSPRODMA are cointegrated, and there is only one cointegrating equation[31]. On the basis of the VEC-Johansen method in table 3, the estimate of the real long run NATREX is NATRJ. Table 4, is based upon a direct OLS method of estimation, since we know that the three variables are cointegrated. The OLS estimate of the real long term NATREX is denoted NATROLS.

Table 3: *VEC Estimates of longer run real NATREX*

Sample: 1971:1 2000:3; ***Included observations:*** 117

Test assumption: Linear deterministic trend in the data

Series: EUUSREDPMA EUUSCRATMA EUUSPRODMA

Lags interval: 1 to 4

Likelihood	**5 Percent**	**1 Percent**		**Hypothe-sized**
Eigenvalue	***Ratio***	***Critical Value***	***Critical Value***	***No. of CE(s)***

[30] The NLS estimates do not make much sense.

[31] The same results are obtained if the logarithms of the variables are used.

0.423891	71.18299	29.68	35.65	None **
0.044501	6.662322	15.41	20.04	At most 1
0.011357	1.336327	3.76	6.65	At most 2

**(**) denotes rejection of the hypothesis at 5%(1%) significance level*
L.R. test indicates 1 cointegrating equation(s) at 5% significance level
Normalized Cointegrating Coefficients: 1 Cointegrating Equation(s)

EUUSREDPMA	EUUSCRATMA	EUUSPRODMA	C
1.000000	4.324569	−5.630790	−0.948782
	(0.93104)	(0.59107)	

Log likelihood 1094.665

Table 4: *OLS estimates on longer run NATREX*
LS // Dependent Variable is EUUSREDPMA

Sample: 1971:1 2000:1; ***Included observations:*** 117 after adjusting endpoints

Variable	Coefficient	Std. Error	T-Statistic	Prob.
EUUSCRATMA	−1.128995	0.549874	−2.053188	0.0423
EUUSPRODMA	1.745565	0.222161	7.857218	0.0000
C	0.815713	0.489005	1.668105	0.0980
R-squared	0.351661	Mean dependent var		1.04489

			1
Adjusted R-squared	0.340287	S.D. dependent var	0.170864
S.E. of regression	0.138780	Akaike info criterion	-3.924417
Sum squared resid	2.195642	Schwartz criterion	-3.853592
Log likelihood	66.56261	F-statistic	30.91702
Durbin-Watson stat	0.039476	Prob(F-statistic)	0.000000

Conclusions. *First:* each method yields the same *qualitative/sign* results. A rise in relative social consumption significantly depreciates the longer run real exchange rate, and a rise in relative productivity significantly appreciates the real exchange rate, as claimed by the NATREX model. *Second:* the *quantitative* values of the coefficients are quite different. Third: Figure 6 showsthat the OLS estimate produces a significantly better fit than does the VEC-Johansen estimate.

Figure 6. Actual real euro EUUSREDPMA OLS estimate NATROLS; VEC estimate NATRJ 4Q moving averages

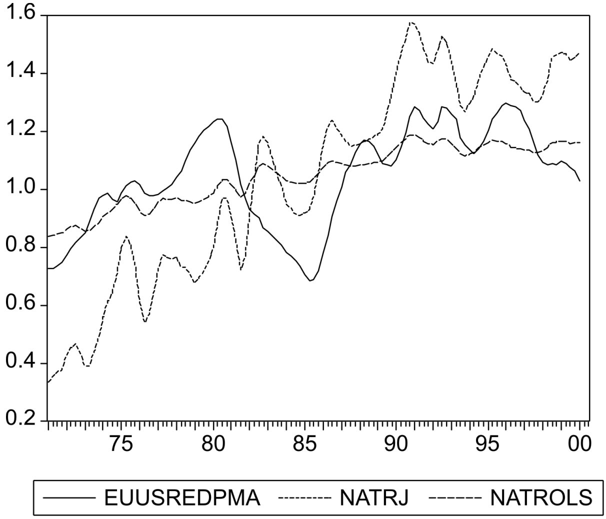

Figure 6

Estimates of the longer-run real NATREX.
R(t) = EUUSREDPMA = actual value real euro ;
NATROLS = OLS estimate; NATRJ =VEC estimate

The medium run real NATREX, variable NATMROLS, in table 5 is based upon equation (16). It is the sum of BZ(t), the longer run NATREX from table 4, and the error correction [R(t-h) - BZ(t-h)]. Since our variables are 4Q moving averages, we use h = 4 in the lag in the error correction. The predicted value of the medium run real NATREX, based upon table 5, is denoted NATMROLS, to indicate that it is real medium run RM and based upon OLS. The values of the coefficients are significant and sensible. The coefficient of the longer run real NATREX is 0.89 with a standard error of 0.09. It is not significantly different from unity. The coefficient of the error correction is 0.74, which indicates stability, and a relatively rapid convergence. The constant is not significant.

Table 5: *NATMROLS, MediumRun NATREX*
LS // Dependent Variable is EUUSREDPMA

Date: 02/21/01 ***Time:*** 10:00

Sample: 1972:1 2000:1

Included observations: 113 after adjusting endpoints

Variable	Coefficient	Std. Error	T-Statistic	Prob.
NATROLS	0.894115	0.091436	9.778635	0.0000
ECROLS(-4)	0.746247	0.062952	11.85417	0.0000
C	0.112140	0.096919	1.157050	0.2498
R-squared	0.685340	Mean dependent var		1.055858

Adjusted R-squared	0.679619	S.D. dependent var	0.163356
S.E. of regression	0.092463	Akaike info criterion	-4.735700
Sum squared resid	0.940438	Schwartz criterion	-4.663291
Log likelihood	110.2270	F-statistic	119.7916
Durbin-Watson stat	0.118678	Prob(F-statistic)	0.000000

5.2. The nominal value of the euro: NATREX and PPP

The logarithm of the nominal value of the euro in equation (17), is the sum of three elements: The logarithm medium run real NATREX minus the logarithm of relative prices in the PPP theory[32], and transitory I(0) differential rates of return on assets.

The transitory factors that are included in the NATREX theory are those that raise aggregate demand: investment less saving or shift the CA function. The SI curve in figure 3 shifts to the left and appreciates the exchange rate from R(0) to R(1). I measure the I(0) factors that shift the SI curve by differential rates of return on investment. The rate of return on investment b(t) is the inverse marginal capital - output ratio. It is b(t) = dY(t)/I(t) = [dY(t)/Y(t)]/[I(t)/Y(t)] = n(t)/j(t), where n(t) is the growth rate of GDP and j(t) is the ratio of investment to GDP. Both b(t) in Europe and b*(t) in the US, and b(t)- b*(t) are I(0) stationary. The differential rate of return EUUSRETURN = [b(t) - b*(t)] is graphed in figure 5.

The estimates of this equation are in table 6. The coefficients have the hypothesized signs and are highly significant. A rise in the medium run real NATREX by 1% appreciates the nominal value of the euro by 1%. A rise in relative EU/US price deflator by 1% unit

[32] See MacDonald and Stein (1999) figure 1, page 4 for a description of the interaction of the real (NATREX) and PPP effects.

depreciates the euro by 1%. A rise in the relative rate of return on investment in Europe relative to the US by 100 basis points appreciates the euro by 28 basis points.

Table 6: *The logarithm of the Nominal Value of the Euro ($US/synthetic euro)*

Log N(t) = a_1 *Log* R_m*[Z(t),t] +* a_2 *log [p(t)/p*(t)] +* a_3 *[b(t)- b*(t)]*

LOGN = C(1) LOGNATMR + C(2)*LOGPPP + C(3)*EUUSRETURN4 + C*

The hypothesis is that $a_1 = 1$, $a_2 = -1$, $a_3 > 0$.

LS // Dependent Variable is LOGN = logarithm of EUUSNERMA;

Sample: 1971:4 2000:1; ***Included observations:*** 114 after adjusting endpoints

Variable	Coefficient	Std. Error	T-Statistic	Prob.
C(1) LOGNATMR	0.964985	0.173830	5.551324	0.0000
C(2) LOGPPP	-1.023130	0.124057	-8.247273	0.0000
C(3) EUUSRETURN	4 0.278595	0.116698	2.387310	0.0187
C(4)	0.009489	0.028273	0.335612	0.7378
R-squared	0.494611	Mean dependent var		0.189247
Adjusted R-squared	0.480827	S.D. dependent var		0.169007
S.E. of regression	0.121776	Akaike info criterion		-4.176690
Sum squared resid	1.631227	Schwartz criterion		-4.080683
Log likelihood	80.31236	F-statistic		35.88466

Durbin-Watson stat	0.201099	Prob(F-statistic)	0.000000

Figure 7 compares the ACTUAL logarithm of the nominal exchange rate with the FITTED value of the nominal exchange rate from table 6. The variables are normalized, so that they are in units of standard deviations. Except for the period of the mid 1980s, the residual is less than 1 standard deviation.

Figure 8 displays the recursive estimates of the coefficients in table 6.The estimated equation is structurally stable from 1985 - 2000. We obtain the same estimates of the coefficients if the sample period is: 1971:4 - 1994:2 or 1971:4 - 1996:4, as we did for a sample period of 1971:4 - 2000:1. That implies that: if we estimated the system during the period 1971:4 - 1994:2 and used these estimates to predict "out-of-sample" 1994:3 - 2000:1, we obtain the same predictions as are described by figure 7.

Figure 7. ACTUAL Logarithm of Nominal NATREX, $US/Euro
FITTED = Estimate of equation (17),based upon table 6.
log N(t) = C(1)*log Rm(Z(t),t) + C(2)*log [p(t)/p*(t)]
+ C(3)[b(t) - b*(t)] + C(4);
RESIDUAL = ACTUAL - FITTED. Normalized variables, 4Q MA.

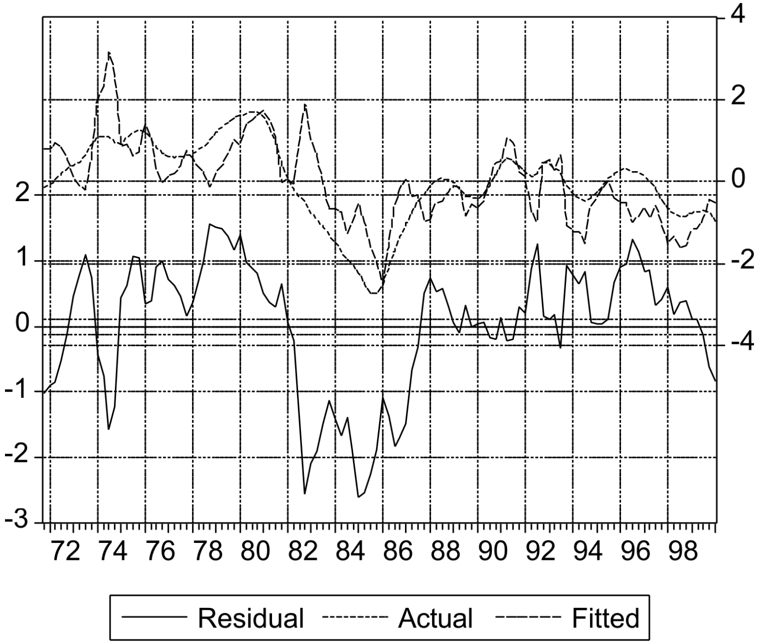

Figure 7
Log nominal exchange rate (ACTUAL),
estimated table 6 (FITTED), residual 4Q moving average

Figure 8. Recursive estimates of coefficients C(1) - C(4) in table 6.

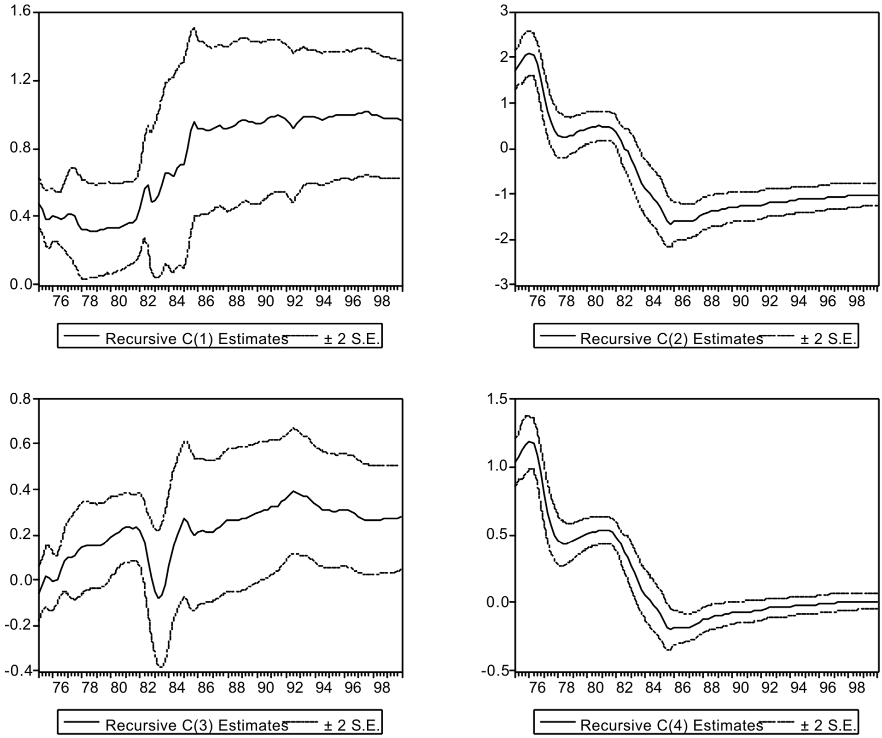

Figure 8
Recursive estimates of coefficients table 6

5.3. The current value of the Euro

An important question is whether the euro is currently overvalued or undervalued. *Our data are 4Q moving averages*. Figure 7 shows that the actual nominal exchange rate converges to the nominal NATREX and that, except for the period of the US bubble in the mid-1980s, the deviation has been less than a standard deviation.

We have stressed that the studies of the reduced form NATREX yield the same qualitative results concerning the signs of the coefficients of our Z(t) fundamentals. The studies obtain different quantitative estimates according to the method of estimation. Therefore, one should not have too much confidence in a number per se, but should associate each number with a range of J. L.

Stein: An evaluation of the Euro 38 uncertainty. In our estimate of the nominal NATREX[33], denoted N*(t), the standard error of the regression is $0.12. Therefore all concepts of "misalignment" should have *a range of doubt* of 1.5 standard errors or $0.18.

Table 7 presents the actual nominal value $US/euro, the estimated nominal NATREX N*(t), the actual real value R(t), the medium run real NATREX $R_m[Z(t)]$, and the differential return on investment [b(t) – b*(t)] for the period 1998:1 - 2000:1. The synthetic nominal euro was overvalued 1995-98. However, by the inception of the Monetary Union 1999:1, the nominal exchange rate was undervalued by $0.04. Our last observation 2000:1 indicates that the actual value of the euro N = $1.03 was undervalued by $0.14 from the estimated value N* = $1.17.

Table 7: *1999:1 - 2001:1. 4Q moving averages*
N(t) = ACTUAL $US/euro
$N^*(t) = 1\, R_m[Z(t)] - 1.4[p(t)/p^*(t)] + 0.34[b(t) - b^*(t)] + 1.4$

	N(t)	N*(t)	R(t)	Rm[Z(t)]	b(t)–b*(t)
1998:1	1.084042	1.100578	1.094195	1.039460	−0.101309
	1.058397	1.107334	1.082823	1.075784	−0.079231
	1.045453	1.096821	1.090243	1.053949	−0.063839
	1.050972	1.100217	1.085213	1.048599	−0.066665
1999:1	1.064213	1.103436	1.097787	1.100458	−0.069837
	1.071049	1.122098	1.090115	1.095234	−0.084981
	1.072262	1.133443	1.075924	1.117903	−0.100581
	1.06635	1.15915	1.06345	1.16770	−0.0911

33 This is the same as table 6, but not in logarithms.

	7	4	2	1	84
2000:1	1.031501	1.170497	1.029596	1.153445	-0.073578

R(t) = real value euro; Rm[Z(t)] = medium run real NATREX; N(t) = nominal NATREX; [b(t)- b*(t)] = differential return on investment. All coefficients, except the constant, are significant at the 1% level. Standard error of regression: $0.12, adj R-squared 0.48.*

There are several possible reasons for the possible "misalignment". First: The $0.14 is less than our *range of doubt.* Second: The longer-run real NATREX (NATROLS in figure 6) has been stable since 1990, and the actual real euro has fluctuated around it. The longer run equilibrium R*(t) = BZ(t)has been stable. The main reason for the recent undervaluation of the nominal rate is that the *real* value R (2000:1) = $1.03 is undervalued by $0.12 relative to the estimated *real* medium run NATREX R_m[2000:1] = $1.15. The *transitory* perturbations, the I(0) factors graphed in figure 5, favored the US. Since 1996, the "new economy" produced a differential rate of return on investment [b(t) - b*(t)] in favor of the US. This factor produces shorter-run depreciation effects. As figure 5 shows, the differential has been stationary around a zero mean[34]. If this differential should lead to a permanent change in relative productivity, then the longer run value of the euro would depreciate. Based upon the information available and the estimation methods, there is no reason to believe that the equilibrium value of the euro has depreciated, and the "undervaluation" is within the *range of doubt.*

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