

# Structural Breaks : Tests of the Present-Value Government Borrowing Constraint and Stability of Debt-GDP Ratio in the UK: 1970-2000

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## Abstract

*In this paper we argue that allowance for structural breaks (Perron, 1989, Zivot and Andrews, 1992) in the relevant time-series can provide more accurate tests of the present-value borrowing constraint (Hamilton and Flavin, 1986) by accommodating any changes in the processes generating budget deficits and alterations in fiscal policies. Our results confirm that the UK government has satisfied the present value constraint during the period 1992-2000.*

**Key Words:** Hamilton and Flavin Stationarity, Structural Breaks, Cointegration Stability Perron Zivot and Andrews

**JEL Classification:** H61 H62 C22 C52 O52

## 1. Introduction

The size of government deficits has been a major concern of the governments in both the developed and the developing countries. Hamilton and Flavin (1986) (HF) proposed a simple test for checking sustainability of fiscal deficits, which has since been applied in the contexts of many countries. The research on fiscal sustainability in the UK prior to HF's was based on conventional econometric

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methods (Akhtar and Wilford, 1979; Cobham, 1980); HF on the other hand, proposed a test of intertemporal budget constraint which lends itself more easily to time – series methods. More recent research on analysis of fiscal deficits in the UK and in the OECD countries have adopted some time-series methods to test the present value government borrowing constraint

(Ahmed and Rogers, 1995, Fève and Henin, 2000, Uctum and Wickens, 2000). But the importance of structural breaks has not been considered in any of these studies.

Akhtar and Wilford (1979) (AW) have used quarterly data for 1963–1976 to assess the influence of the UK's fiscal deficits on the money stock. Data have been split to take the influence of competition and credit control into account during the period, 1971–1976. They reach the conclusion that the public sector deficit has been an important influence on the money stock and that this influence was growing in the recent years. Cobham (1980) has criticised AW's work on the grounds of lack of clear interpretation/definition of various measures such as “domestic credit”, “passive monetary policy” etc., and their use of non-seasonally adjusted data and after making corrections for these, Cobham rejects AW's conclusion that the effectiveness of monetary policies critically depends on complementary fiscal policies. Ahmed and Rogers (1995) have used long-run historical data from 1700 and found strong evidence in favour of fiscal sustainability in the US and some evidence on fiscal sustainability in the UK. Uctum and Wickens' (2000) (UW) study is on the eleven OECD countries including the UK and on the US. To examine fiscal solvency they have conducted “Integrability Tests” with augmented Dickey –Fuller (ADF) (Dickey and Fuller, 1976) and Phillips–Perron (PP) (Phillips and Perron, 1988) statistics; understandably, not much detail is provided on the UK economy as such; UW have modified HF (1986) method combining the infinite and the finite horizon approaches together. This is to take account of the fact that to attain and/or maintain sustainability, it may be ne-

cessary to alter fiscal policy itself. We argue here that any changes in fiscal policy will have been included in the data on debt, fiscal deficits/surplus or even in rates of interest and an unbiased search for structural breaks in these series (Zivot and Andrews, 1992) will show if changes in policy had significant implications for fiscal sustainability in the UK.

In this paper, therefore, we propose an extended time-series econometric method of testing HF's present value government borrowing constraint and stability of debt–GDP ratio, and show that our extended method will take care of the criticism of HF's raised by UW. The HF method has been criticised by UW on the ground that the method assumes that the processes generating deficits and debt will continue to be the same in future. We discuss below how this shortcoming of HF's method can alternatively be tackled by a thorough time-series analysis of an unbiased search for structural breaks. An unbiased search for breaks lets the data tell the story. The data on relevant series must include the effects of any alterations in fiscal policy undertaken by the government. The method of endogenous time of break (Zivot and Andrews, 1992) will be able to show when any alterations in fiscal policy have been significant enough to make a change in fiscal stance or in the present value government borrowing constraint. So far, application of time-series methodology to the issue of budget deficits in the UK has been limited to checking stationarity by augmented Dickey–Fuller (ADF) tests (UW, 2000) and by modified ADF tests (Bohn, 1995, Fève and Henin, 2000). We show that a more complete time-series analysis of stationarity and cointegration properties allowing for structural breaks would be a more reliable test of fiscal sustainability. Such detailed time-series analysis allowing for structural breaks, we believe, can throw more light on the issue of budget deficits in the UK and, to our knowledge, such detailed analysis has not been adopted for the UK economy so far. We have the following objectives:

- (1) We follow Hamilton and Flavin's (1986) present – value borrowing constraint approach to show how a link can be developed between the present-value borrowing constraint and stationarity of the relevant time-series;
- (2) We extend Hamilton and Flavin's test to develop the relation between the present value constraint and cointegration of the relevant time –series;
- (3) We check the stationarity and cointegration properties using annual data for the UK economy for the period, 1970--2000 and
- (4) Extend stationarity and cointegration tests to make allowance for structural breaks at the known and the unknown points; and finally
- (5) We show that the method of locating the time of break determined by the data themselves makes HF's test of fiscal sustainability more reliable. While steps (1) to (3) have been covered in one form or other in some of the studies mentioned in the above discussion, tests of stationarity and cointegration properties under structural changes have not been adopted so far<sup>1</sup>. We argue that allowance for breaks in government deficit/surplus, government debt and all the other relevant series at known and unknown time-points (Perron, 1989, Zivot and Andrews, 1992) can take into account any possible changes in the "processes generating deficits and debt"<sup>2</sup>. Therefore, tests of stationarity and cointegration allowing for structural changes would be more reliable for examining inter-temporal budget balance and stability of the debt-GDP ratio in the UK economy during the period under consideration.

The plan of the paper is as follows:

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<sup>1</sup> <sup>1</sup> Duck (1992) has tested a number of macroeconomic time-series in the UK for stationarity by Perron's (1989) method only.

<sup>2</sup> <sup>2</sup> UW (2000, p.198) point out that HF's (1986) method cannot take care of such changes

Section 2 develops the theoretical issues: how intertemporal balance of government budget and stability of debt–GDP ratio (HF, 1986, Gupta, 1992) are linked with the stationarity and cointegration properties of government debt and government deficits, the stationarity and cointegration properties of the real interest rate, the deficit–GDP ratio and with those of the growth rate of the economy.

Section 3 includes a brief discussion of the sources of data and our chosen econometric methodologies (Dickey and Fuller, 1981, Nelson and Plosser, 1982, Perron, 1989 and Zivot and Andrews, 1992, Engle and Granger, 1987, Ghatak, 1998). In searching for breaks we follow two methodologies—one on prior assumption of the year of break (Perron, 1989) and the other on an unbiased search for breaks (Zivot and Andrews, 1992).

In Section 4, we interpret the results of tests and argue that the allowance for breaks in the series will accommodate any changes in the processes of deficits and debt generation and will, therefore, make HF's test stronger and more reliable. Estimations in this paper are made using microTSP version 7 and MICROFIT 4.0.

Section 5 provides a summary of the main conclusions of our study.

## **2. Economic Theory and Time–Series Properties of Relevant Variables**

### **The Present–Value Government Borrowing Constraint**

HF (1986) consider intertemporal balance of government deficits and check whether the government faces a present–value borrowing constraint. If the government can not satisfy this constraint, then government deficits exclusive of interest payments are not feasible. The question has profound macroeconomic implications. If deficits are to be financed by future tax increases, then there are no stimulative effects on aggregate demand but there could be strong distortionary and disincentive effects of higher taxes; if de–

ficits are financed out of new money, then such deficits can lead to inflation as corroborated by the experience of many developing countries (Gupta, 1992). The theoretical question whether a permanent budget deficit can be sustained does not have a straight forward answer but a link can be indicated between the interest rate at which the government has to borrow and the rate of growth of the economy. If the real interest rate at which the government has to borrow, is greater than the growth rate of the economy, then, permanent deficits are infeasible and if the real rate of interest is less than the growth rate of the economy, then, such deficits could be feasible. In the latter case, the ratio of debt to GDP does not increase, so, deficits could be tolerated. However no consensus is available in the literature on these theoretical issues and the importance of empirical evidence has been emphasised to sustain either of these theoretical positions (Aschauer, 1985, Barro, 1984, Evans, 1985).

HF propose a direct test of the present-value government borrowing constraint. If this constraint is violated, then perpetual primary deficits<sup>3</sup> meaning deficits excluding interest payments on bonds, are not feasible. The final estimating version of intertemporal government budget constraint can be derived step by step as follows:

First HF collect all government debt of a given coupon and maturity into group  $j$ ;  $d_{j,t}$  is nominal market value of such debt at the end of period  $t$  and  $\theta_{j,t}$  the total nominal coupon payments between  $t-1$  and  $t$ ; supposing no new bonds are issued or redeemed during period  $t$ , changes in the market value of group  $j$  debt can be evaluated using a simple term-structure argument. The real excess one-period holding yield of  $j$ -group bonds relative to the average earned on a comparable one-period bonds, say,  $v_{j,t}$ , then can be written as:

$$v_{j,t} = (d_{j,t} + \theta_{j,t})/P_t - (1+r)d_{j,t-1}/P_{t-1} \dots \dots \quad (1i)$$

<sup>3</sup> <sup>3</sup>

Official deficits include interest payments on government bonds.

where  $P$  is an aggregate price index and  $r$  the real ex post rate of interest on one-period bonds. If real interest rates were white noise or constant and if the expectations theory of term-structure held, then  $E_{t-1} v_{j,t}$  would be equal to 0; in general, positive values of  $v_{j,t}$  mean bond holders have made a capital gain on long-term debt or that short-term rates are higher than average.

The real market value of debt held by the public is:

$$\sum d_{j,t} = B_t \dots \quad (1ii)$$

and this will also change because of official deficit, government spending plus interest payments to public minus tax revenue, all in real terms :

$$df = G_t + R_t - T_t \dots \quad (1iii)$$

and because of changes in the stock of high-powered money in real terms,

$(M_t - M_{t-1})/P_t$ . Taking into account all these components and a random error term, one can write:

$$B_t = (1+r) B_{t-1} - \sum \theta_{j,t}/P_t + \sum v_{j,t} + G_t + R_t - T_t + \Delta M_t / P_t + u_{1t} \dots \dots (2)$$

where  $u_{1t}$  is the error term and it is added to equation (2) to take account of imperfections in the method measuring real government debt and because bond purchases and sales towards the beginning of the period would take place at prices closer to that of  $B_{t-1}$  rather than  $B_t$ .

Also, the interest payment on debt can be written as:

$$R_t = \sum \theta_{j,t} / P_t + u_{2,t} \dots \quad (2i)$$

Therefore, HF write

$$B_t = (1+r) B_{t-1} - S_t + V_t \dots \quad (3)$$

Where the budget surplus,  $S_t$  is defined as

$$S_t = T_t + (M_t - M_{t-1})/ P_t - G_t \dots \quad (3i)$$

and 
$$V_t \equiv \sum v_{j,t} + u_{1t} + u_{2t} \dots \quad (3ii)$$

The formulation (3) has three special features:

- (a) It describes the market value of government debt, not the par value;
- (b) the measure of surplus in equation (3) excludes interest payments from government spending and
- (c) it adds money seigniorage to taxes as a source of revenue for retiring government debt.

By repeated substitution, one can derive the final implication of equation (3) as:

$$B_t = \sum [(S_i - V_i)/(1+r)^{i-t}] + (1+r)^t B_n / (1+r)^n + u_t \dots \quad (4)$$

Where summation runs from  $t+1$  to  $n$ . Equation (4) is the least controversial one as it follows from definitions of fiscal and monetary policy. What should be put to empirical test is what creditors *expect to happen* to the second term in (4),  $(1+r)^t B_n / (1+r)^n$ , as  $n$  gets large.

Letting  $E$  denote the expectations operator and assuming that expectations are formed at date  $t$ ,

HF derive the hypothesis that the government faces a present value borrowing constraint:

$$H_0: B_t = E_t \sum [(S_i - V_i)/(1+r)^{i-t}] \dots \quad (5i)$$

The constraint stated in (5i) is equivalent to the restriction that the real supply of bonds held by the public is expected to grow no faster on average than the rate of interest:

$$H_0: E_t \lim B_n / (1+r)^n = 0 \dots \quad (5ii)$$

A few important implications can be drawn from the pair of equations (5i) and (5ii). They are consistent with a permanent government deficit inclusive of interest rates, that is, a permanent official deficit. Let this constant deficit be  $k = -S + V + rB_{t-1}$ . Then equation (3) implies that



$B_n = nk + B_0$  and that the limiting value of  $B_n/(1+r)^n = 0$  at infinity. Thus, a policy of keeping the interest component of deficit from rising, will eventually force the government to pay off its debts in present-value terms. On the other hand, a permanent deficit exclusive of interest payments,

$-S_t + V_t = k$  is not consistent with (5); for then,  $B_n = k[(1+r)^n - 1]/r + (1+r)^n B_0$  and the limiting value of  $B_n/(1+r)^n = k/r + B_0$  (McCallum, 1985)

So, HF prove that the null hypotheses that the government faces a present-value borrowing constraint or that the budget must be intertemporally balanced are given in (5).

The next stage is the framing of alternative hypotheses: the government deficits (that is negative values of  $S_0$ ) need not be balanced with future surpluses. One interesting class of alternative hypotheses is that

$$E_t \lim [B_n/(1+r)^n] = A_0 > 0 \dots \quad (6i)$$

This alternative hypothesis allows the possibility that a certain amount of real government expenditures,  $r(A_0 - B_0)$ , need never be paid for with taxes. Using this in the equation (4) HF then obtain

$$B_t = E_t \sum [(S_i - V_i)/(1+r)^{t-i}] + A_0 (1+r)^t \dots \quad (7ii)$$

as a general class of solutions to (3). The null hypothesis,  $H_0$  that the government budget must be balanced in present-value terms holds true if and only if  $A_0 = 0$  in equation (7ii)

Equation (7ii) is mathematically equivalent to the models of self-fulfilling fads or speculative bubbles first explored by Flood and Garber (1980).

From above theory, tests of the null hypothesis of the present-value government borrowing constraint have been translated into tests of *stationarity and cointegration* as understood in the time-series methodology.

For developing the link between the present-value government borrowing constraint and stationarity and cointegration properties of

time-series, we need to recall the contents of  $V$ , which includes random error terms to capture creditor's expectations, short-term low interest rates and measurement errors. We may rewrite the final version of the constraint (7ii) as:

$$B_t = E_t \sum [S^*_i / (1+r)^{t-i}] + A_0(1+r)^t + u_t \dots \quad (8)$$

Where  $S^* = S - \sum v_{j,t}$ , and  $u$  is the composite random error term including  $u_{1,t}$  and  $u_{2,t}$ . The present-value borrowing constraint can now be verified by testing whether  $A_0 = 0$  in equation (8). If  $u_t$  and the surplus series are stationary, and if  $A_0$  is 0, then the series  $B_t$  will be stationary and the present value borrowing constraint will be satisfied. Therefore, tests for stationarity for the debt and the surplus series can be used as tests for the present value borrowing constraint. If, however,  $A_0$  is not equal to 0, then  $B_t$  is nonstationary and the present value constraint will not be satisfied. It should also be noted that  $v_{j,t}$  uses a constant interest rate but this assumption may not hold in reality, in which case, stationarity of interest rate also has to be tested as well as the debt and the surplus series for the purpose of testing the present-value borrowing constraint.

If the debt and the surplus series are nonstationary, then  $A_0$  is not equal to zero and, therefore, the present-value borrowing constraint is not satisfied. This is the context where an extension to include the cointegration properties of variables can help<sup>4</sup>. Even if the debt, surplus and interest rate series are nonstationary, the constraint can be satisfied if the debt, surplus and interest rate series are cointegrated despite being nonstationary (Engle and Granger, 1987). Cointegration of nonstationary variables implies

<sup>4</sup> HF themselves have applied the Dickey-Fuller test for Unit Roots (Dickey and Fuller, 1981), generalised Flood-Garber (GFG) test and restricted Flood-Garber test (RFG) (Flood and Garber, 1980). The first one is for testing stationarity, the GFG is for testing stationarity of the random error term,  $u$ , and both the GFG and RFG tests are necessary when expectations of future surpluses are conditioned in part on past surpluses. The GFG tests are very similar to a dynamic linear regression (Alogoskoufis and Smith, 1995) except in the term  $A_0(1+r)^t$

that there exists a linear combination of the nonstationary variables, which is stationary. This implies, in other words, that in the regression equation (8),  $u_t$ , is stationary. So, cointegration tests involving debt, interest rate and surplus variables could also throw light on the null hypothesis of the present value borrowing constraint.

Cointegration tests side-track the problems which could be caused if  $A_0$  were random.

### **Stability of Debt–GDP Ratio**

Stability of debt–GDP ratio is an issue closely related to that of the overall sustainability of fiscal deficits; if real interest rate at which the government has to borrow is less than the rate of growth of real GDP, then deficits are feasible. A permanent deficit inclusive of interest payments can be consistent with the optimising behaviour of bondholders but a permanent deficit exclusive of interest payments is not consistent with such optimising behaviour (McCallum, 1985). A constantly increasing stock of debt does not violate the optimising behaviour as long as the rate of increase is less than the government’s borrowing rate (HF, p.811).

The relation between debt and surplus – GDP ratios and interest rate can be derived easily as follows<sup>5</sup>:

$$\Delta B_t = G - T - \Delta M/P + rB_{t-1} \dots \quad (9i)$$

which can be rewritten as:

$$B_t = (1 + r)B_{t-1} - S_t \dots \quad (9ii)$$

Dividing through by GDP denoted by  $Y_t$  and writing  $Y_t = (1 + g)Y_{t-1}$ , where  $g$  is the rate of growth of real GDP, (9ii) can be rewritten as:

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<sup>5</sup> <sup>5</sup> In deriving the following equations (9i) – (13), we have deviated from the continuous formulation used by HF and we have not used any of the restrictive assumptions of Gupta (1992, p. 38). Gupta approximated  $(1+r)/(1+g)$  by  $(1+r-g)$  and assumed zero monetisation of deficits.

$$B_t/Y_t = [(1+r)/(1+g)] B_{t-1}/Y_{t-1} - S_t/Y_t \dots \quad (9iii)$$

Denoting the ratios by respective lower case, we can rewrite the non-homogeneous first-order difference equation in debt-GDP ratio,  $b$ , as:

$$b_t = [(1+r)/(1+g)] b_{t-1} - s_t \dots \quad (10)$$

The equilibrium/steady state value<sup>6</sup> is

$$b^* = -s_t / [1 - (1+r)/(1+g)] = -s_t(g-r)/(1+r).. \quad (11i),$$

which implies that  $b^*$  is stable only if rate of growth of real GDP,  $g$ , is greater than the real rate of interest,  $r$  and  $b^*$  is unstable if  $g$  is less than  $r$  (Yellen, 1989). When  $g$  is less than  $r$  but there is a budget surplus there will be two opposing tendencies operating on the debt/GDP ratio. Higher  $r$  will tend to raise  $b$  and the budget surplus will tend to reduce  $b$ . The net outcome will depend on the relative strength of the two tendencies.

The general solution to the dynamic equation (10) is

$$b_t = b^* + (b_0 - b^*) [(1+r)/(1+g)]^t. \quad (12)$$

where  $b^*$  has been derived in (11i) and  $b_0$  is the initial value of the debt-GDP ratio assumed to be known.

For the purpose of exploring time-series properties and deriving econometric tests for stability of debt-GDP ratio, we can write the stochastic form of equation (10):

$$b_t = [(1+r)/(1+g)] b_{t-1} - s_t + \varepsilon_t \dots \quad (13)^7$$

where  $\varepsilon_t$  is the random error.

The debt/GDP ratio will be stable if  $b$  and  $s$  are stationary. If  $b$  and  $s$  are nonstationary but  $b$  and  $s$  are cointegrated, then there would be an equilibrium linear combination of  $b$  and  $s$  which will be stationary.

<sup>6</sup> <sup>6</sup> This is derived by assuming  $b_t = b_{t-1} = \dots = b^*$

<sup>7</sup> <sup>7</sup> All the formulations from (9i) - (13) treat  $r$  as a proportion, not as percentage.

### **3. The Time–Series Methodology Applied in the Paper**

#### **Tests of Stationarity**

We will test for stationarity with and without allowance for structural breaks. Tests for stationarity will be made by the usual augmented Dickey –Fuller (ADF) t- (Dickey and Fuller, 1976). The Mckinnon critical values of ADF-t are given automatically on the TSP 7 output. We will choose the length of lag, k, according to the Ng and Perron (1997) criterion for all the ADF and the DF-t tests. The ADF t-values are given in Table 1A and they will be interpreted in the following section.

#### **Stationarity and Structural Breaks**

Nelson and Plosser (1982) applied the ADF tests for stationarity to a number of macroeconomic time-series in the USA and obtained the sensational conclusion that all but the unemployment series had unit roots and that they were stationary in their first difference. This conclusion was challenged by Perron (1989, Zivot and Andrews, 1992, Andrews, 1993) and others who consequently established that the ADF tests could be biased in favour of acceptance of the URH if possible structural breaks are ignored in the series. In long run time-series data, therefore, tests for stationarity should pay due regard to possible changes in the level and slope of the trend and/or rate of growth in the series. The Perron and the Zivot and Andrews' methods involve introducing dummy variables at exogenous and endogenous break points respectively and their tests resulted in a number of cases in rejection of the URH in Nelson and Plosser's data. Structural breaks can occur in a series on account of various changes taking place in the economy due to domestic or external factors. In the present context, such changes in the pattern of growth of a relevant series can occur also due to changes in the processes of deficits and debt generation. Analysis of endogenous breaks (ZA) will accommodate such changes in the

processes of debt and deficit generation; on the other hand, analysis of exogenous breaks (Perron, 1989) at known time points, such as the oil price shock in the seventies or the inception of the EMS in 1979 etc. will reflect the reaction of all series to these major events. We have applied the extended tests of stationarity using Perron's and ZA's methodologies and these tests are more reliable in examining fiscal solvency. Both these methods need inclusion of various dummy variables<sup>8</sup>.

Structural breaks can imply a change in the level of trend, or a change in the slope of the trend or a change in both. In the present context, if the relevant series are nonstationary when tested without allowance for break, it does not necessarily follow that the present value borrowing constraint has not been satisfied or that the debt-GDP ratio has been unstable for the entire period in question; if the relevant series are stationary after breaks are allowed for, then it means that the constraint is satisfied after the period of break and that fiscal policy has been sustainable from that period on. In other words, stationarity, if attained after the break year, it should imply that the government is taking the right steps and going in the right direction to put fiscal house in order and that government deficits, debts, etc., are undergoing significant changes after respective break year to maintain feasibility and solvency of fiscal stance.

### **Tests of Cointegration(CI)**

We will test for cointegration (CI) using Johansen's method (Johansen, 1988). The method involves estimation of vector autoregression(VAR) models for various lags and using the likelihood ratio(LR) tests based on the maximum Eigen value and based on the trace of the stochastic matrix of coefficients of the VAR. These LRs test the null hypotheses of the number of CI vectors (nc) against

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<sup>8</sup> <sup>8</sup>

Details of these equations are available in Ghatak (1997)

respective alternative hypotheses, the bone of contention being uniqueness of CI vectors. We have reported the LRs and the respective critical values at the 5% and 10% levels of significance for the Trace test for the null hypothesis of the number of cointegrating vectors (Johansen, 1988). Johansen's tests are specially recommended for multiple cointegration when Engle and Granger's two-stage test (Engle and Granger, 1987) or the simple test by the cointegration regression Durbin Watson statistic are not sufficient (Engle and Yoo, 1991). Most of the required critical values will be reported in Aides to respective tables and in various parts of Table 2.

We will extend CI tests to include structural breaks, if evidence of such breaks were found in our prior tests for stationarity.<sup>9</sup> As has been already established ignoring structural breaks in series may bias tests of stationarity in favour of accepting the null hypothesis of unit roots. Tests of cointegration are also tests of stationarity of residuals from cointegration regression (Engle and Granger, 1987); therefore, it is important to make allowance for structural breaks in tests of cointegration as well, to derive unbiased results. The issue of structural breaks has been overlooked in the tests for cointegration and existence of long-run equilibrium relation has been assumed away. To pre-empt this possibility, one can add dummy variables for intercept and slope changes for structural breaks at the known and unknown time points Ghatak (1998). The details of the slope and intercept dummy variables are provided in table 2B. Once the dummy variables are added, tests for cointegration can be carried out by the Johansen method in the usual way.

### **Data and Definitions of Variables**

The variable, government surplus, in real terms, has been defined in equation (3i) above.

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<sup>9</sup> <sup>9</sup> Cointegration tests in the presence of structural breaks have been discussed in very much detail in Ghatak (1998)

This definition takes account of the fact that government deficit could be financed by changes in money creation as well as new debt issue. HF (1986, p. 812) give the details of how the surplus and debt variables should be defined for the purpose of testing the present value constraint. We checked these details in the Financial Statistics, April, 2001 (Patel, CSO) and constructed two measures of government surplus— one including and the other excluding money seigniorage. The latter will be denoted by the symbol, NS and surplus excluding money seigniorage by S. For the sake of convenience, in all our theoretical discussion above, we have kept only the one symbol, S, for surplus including money seigniorage. We have used the first difference of seasonally adjusted M0 series for measuring money seigniorage. A comparison of results for the two definitions of government surplus can provide insights into the importance of money financing of deficits in the UK.

We have used annual data for the UK economy covering the period 1970 – 2000. The data on government spending and government revenue are available for longer periods but those on government debt, on interest payment on debt are available only from 1970. We have used two sources of data— the International Financial Statistics Yearbook, 1999 and Financial Statistics, April, 2001. All the relevant series are in 1985 prices. There are two rates of government bond yield –short-run and long-run The real rate of interest has been constructed by deducting rate of inflation from short-run and long-run bond yield denoted respectively as  $r_s$  and  $r_l$ ; also following UW (2000) a real rate of discount has been constructed denoted respectively as  $r_s^*$  and  $r_l^*$  by deducting rate of growth of real GDP from  $r_s$  and  $r_l$ . According to UW, this is the effective rate of interest which should be used for checking stationarity of the discounted debt series.

For applying Perron's method of exogenous structural breaks, the assumed years of breaks for the UK economy will be taken to be the year of the break down of the Bretton Woods system and the



year of oil price shock, 1973, the inception of the EMS, 1979, brief entry into the ERM, 1990, and the year of ERM crisis culminating in the rejection of the Maastricht treaty, 1992.

To apply ZA's method, as already explained above, we search for the break year continuously from 1971 to 1999 and the year producing the minimum ADF-t for a series is taken to be the year when patterns of growth of the series underwent a change. This method of searching for breaks is unbiased and superior to Peron's method as it reduces the probability of Type II error (Ghatak, 1997). If all the relevant series retain their stationarity and cointegration properties even after allowing for structural breaks preferably by ZA's method, we will accept that as stronger evidence in favour of the present -value borrowing constraint and in favour of stability of the debt-GDP ratio.

#### **4. Interpretation of Results**

The most important aspect of the results given in Table 1A are that the debt series is stationary. This is in favour of the null hypothesis that the UK government satisfies the present value constraint meaning that fiscal policy has been sustainable. Now to check whether the test of stationarity is biased because possibility of breaks was ignored, we look at the results in Table 1B and confirm that the debt series is stationary even after allowing for a break at 1973, at 1979, at 1990 and at 1992. On an unbiased search for breaks, the debt series is stationary after allowing for a break in 1989, in the year before entering the ERM. The same conclusions hold for the debt-GDP ratio. The surplus series including the money seignorage and without the money seignorage are both nonstationary but they are integrated of order 1, when no allowance is made for breaks. This result is not sympathetic to the null hypothesis of the present value constraint; but after allowing for breaks in the level and slope of the trend, surplus including seignorage is stationary at the 5% level after 1979, 1990 and 1992 and at the 10%

level after allowing for breaks in 1973. After 1992, the surplus series including changes in high-powered money is stationary. These are very supportive of the null hypotheses of the government meeting the present value constraint. As already explained, for testing the null hypothesis of the present value constraint government surplus series should be properly defined—excluding interest payment and including changes in high-powered money because fiscal deficit can be financed by both new debt and new money. The differences in the nature of the surplus series with money seigniorage from those of the surplus series without, demonstrate that the UK government have used redemption of debt through money seigniorage. Real interest rates as measured by the short- and the long-run government bond yields,  $r_s$  and  $r_l$  and the rates of discount based on these two rates,  $r_s^*$  and  $r_l^*$  are all integrated of order one and they all stay nonstationary if we search for breaks and stop when the ADF-t is minimised. However, they become stationary if a break is allowed for at 1979, the year of inception of the EMS. As the rates of interest are nonstationary, this result is not supportive of the null hypothesis that the present value constraint is honoured. This is the stage when cointegration properties of the three variables, debt, surplus and real interest rate have to be checked. Cointegration of debt, surplus including money supply changes and real interest rate series is needed to confirm that the present value constraint is satisfied. Because if variables are cointegrated despite being nonstationary, then there exists an equilibrium linear combination of them which is stationary (Engle and Granger, 1987). The surplus-GDP ratio,  $ns$ , ( $=NS/GDP$ ) is stationary in first difference only at the 10% significance level and nonstationarity is maintained even after allowing for breaks. The latter finding makes tests for cointegration of  $b$ ,  $ns$ ,  $r_s$  (or  $r_l$ ) and  $g$  crucial for determining stability of the debt-GDP ratio. None of the PP statistics are statistically significant at the 5% level but this is not surprising in data series of only 31 observations and it is the ADF-t values

which are more important in tests for stationarity.

The results of tests of cointegration are given in Table 2. Part A reports results without allowing for structural changes. The debt, surplus including (and without) changes in high-powered money and real interest rate (short and long run) are all cointegrated at 5% significance level by Johansen's trace tests. We report these trace test statistics to establish uniqueness of cointegration vectors for only the optimum lag. The null hypothesis of no cointegration vector,  $nc=0$ , is rejected in all four cases against the alternative hypothesis of one cointegration vector,  $nc=1$ . This is evidence in favour of the hypothesis that the government satisfied the present value borrowing constraint during the period 1970–2000. Debt–GDP ratio, surplus–GDP ratio, rate of growth of real GDP and real long-run yield are cointegrated at 10% level by the trace LR test but at the 5% level by the LR test for maximum Eigen value for short-run interest rate; they are cointegrated at the 5% level by the Trace LR test when long-run interest rate is used. The combined evidence that there exist structural breaks in individual series and that there exists cointegration among the series when grouped together imply possible co-breaking.<sup>10</sup>

However, the next step is to check whether cointegration is maintained even after allowing for structural changes in the years of significant breaks found in the respective series.

These CI tests statistics are presented in part B of Table 2. The CI regression with structural changes include the dummy variables for assessing stability of cointegration vectors. They are explained in respective columns of Table 2 with reference to the year of break found in the previous tests of stationarity. The results confirm existence of stable and unique cointegration vectors at the 5% or 10% levels by the Trace LR test in all cases. Uniqueness of CI

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<sup>10</sup> 10

We are grateful to a referee for pointing our attention to this.

vectors is ensured by the LR test for maximum Eigen value at higher levels of significance in all cases.<sup>11</sup>

Finally, it should be noted further that among the cointegrated vectors, only the B and b series have proved to be integrated of order 0  $I(0)$  in all the tests carried out in our paper but the rest of the variables/series, which are considered in our paper as explanatory variables in the cointegration regressions have shown unit roots and, therefore, have proved to be integrated of order 1  $I(1)$  in most cases. This enables us to escape the danger of misreading stationarity of individual series for cointegration ; therefore, in turn, the evidence obtained in our paper on cointegration of the series included in each relevant regression is not a mere reflection of stationarity of individual series but it is a genuine reflection of cointegration of these time – series.<sup>12</sup>

## 5. Summary and Conclusions

The stationarity of the debt series, B, throughout the period with and without breaks demonstrates strong evidence that the budget has been in balance throughout the period under our consideration, 1970–2000 and that the government effort to satisfy the present value constraint has been particularly strengthened after 1989; the evidence in favour of fiscal prudence is further confirmed in our findings by stationarity of the surplus series including changes in money supply. Perron's tests reveal evidence of significant structural breaks at crucial years, viz., 1973, 1979, 1990 and 1992 in all three series. The latter set of results can be taken to imply that fiscal policies were under scrutiny and alterations were made to ensure sustainability of budget deficits. According to ZA's unbiased tests for endogenous breaks, the debt series, the surplus series including money seigniorage and the debt-GDP ratio

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<sup>11</sup> <sup>11</sup> The LRs for maximum Eigen value for various null hypotheses on the number of CI vectors are not reported in the Tables but are available from the author on request.

<sup>12</sup> <sup>12</sup> We are grateful for a second referee's comment in this context.

series are stationary. Considering that ZA’s tests reduce the probability of accepting a false null hypothesis, we safely reach the following conclusions about fiscal sustainability in the UK: that the UK government has satisfied the present-value borrowing constraint after the year 1992 and that the debt–GDP ratio has been stable during the period 1992–2000. These conclusions are also confirmed by the cointegration properties of all the relevant variables. As most of the series considered as explanatory variables in the cointegration regression are I(1), the evidence on cointegration is not a mere reflection of stationarity of individual series but of genuine cointegration. All our results are subject to rather a small number of observations, namely 31. As already pointed out, this is mainly due to the fact that the data on government debt series are not available for a longer period than we have used in the paper. This shortcoming can hopefully be alleviated in future research.

**Table 1A:** *Results of Tests for Stationarity*

Series	ADF–t	Series	ADF–t	Series	ADF–t	Series	ADF–t
B	– 7.7026* *	Rs	–2.7441	Δrs	– 5.4515**	rl	–2.8488
S	–2.8799	ΔS	– 4.3255* *	b	– 13.5912* *	g	– 3.6911*
Δrl	–4.6473	rs*	–2.3255	Δrs*	– 4.5357**	rl*	–2.6186
NS	–2.8294	ΔNS	– 6.6841* *	ΔM	– 58.2377* *	ns	–2.81
Δrl*	– 4.4713*	Δns	–3.41 <sup>+</sup>				

\*

**Aide to Table 1A:**

5% Critical Values of ADF-t for 31 observations for variable and first difference

	lag order 1	lag order 2
Variable	-3.5731	-3.5796
First Difference	-3.5796	-3.5867

**Table 1B:** Results of Stationarity Tests including Structural Breaks (Perron's Method)<sup>1</sup> Values of TB and  $\lambda$  Respectively for Series

Series	1973 and 0.13		1979 and 0.32		1990 and 0.68		1992 and 0.74	
	ADF-t	PP	ADF-t	PP	ADF-t	PP	ADF-t	PP
<b>B</b>	-	-	-	-	-	-	-	-
	5.07*	18.9	6.96*	22.9	11.45*	30.50	10.40*	28.0
<b>S</b>	-1.35	-	-1.24	-	-3.30	-8.89	-2.06	-
		5.51		5.07				8.41
<b>NS</b>	-	-	-	-	-	-	-	-
	3.55 <sup>+</sup>	14.9	4.61*	19.6	6.32**	25.87	5.87**	27.8
<b>rs</b>	-2.94	-	-	-	-3.48	-	-3.47	-
		15.5	4.45*	22.7		18.25		18.0
<b>rl</b>	-3.11	-	-	-	-3.54	-	-3.56	-
		18.0	4.55*	22.9		19.33		19.4
<b>b</b>	-	-	-	-	-18.12	-	-17.49	-
	8.57*	23.9	11.94	28.6		32.19		31.2
<b>g</b>	-	-	-3.74	-	-3.78	-	-3.77	-
	3.71 <sup>+</sup>	20.1		21.7		24.59		22.5
<b>rs*</b>	-3.67	-	-3.31	-	-2.60	-	-2.58	-
		25.5		25.5		18.68		17.8
<b>rl*</b>	-	-	-3.55	-	-2.77	-	-2.85	-
	4.77 <sup>+</sup>	33.6		26.2		19.99		20.1

		3		3			5
ns	-2.89	-	-2.58	-	-5.21*	-	-2.69
		15.9		9.06		15.49	13.1
		5					8

1. Equation Estimated:  $X_t = a_1 + a_2X_{t-1} + a_3\Delta X_{t-1} + bt + c_1D_{1t} + c_2D_{2t} + c_3D_{3t} + u_t$

Where the dummy variables are defined as :

$D_{1t} = 1$  for  $t > T_B$  and 0 otherwise;  $D_{2t} = 1$  for  $t = T_B + 1$  and 0 otherwise;  $D_{3t} = t$  for  $t > T_B$  and 0 otherwise.

$\lambda = t/T_B$

**Aide to Table 1B**

Perron's Critical Values for ADF-t and PP for values of  $\lambda$

Critical values of ADF-t and PP respectively

Values of  $\lambda$

Significance level	0.2	0.3	0.4	0.5	0.6					
1%	-4.65	-39.97	-4.78	-42.98	-4.81	-45.52	-4.9	-44.07	-4.88	-44.75
5%	-3.99	-29.95	-4.17	-32.47	-4.22	-33.22	-4.24	-33.79	-4.24	-33.19
10%	-3.66	-25.50	-3.87	-27.9	-3.95	-29.39	-3.96	-29.41	-3.95	-29.04

Source: Perron (1989, p.1377)

**Table 1C:** Results of Stationarity Tests including Structural Breaks (ZA Method)<sup>2</sup>

Series	ADF-t (min)	TB	Series	ADF-t(min.)	TB	Critical Value
B	-12.597**	1989	rs	-4.72	1979	1%: -5.57
S	-4.07	1974	NS	-6.005**	1992	5%: -5.08
B	-20.39**	1989	g	-4.44	1989	10%: -4.82
rl	-4.74	1980	rl*	-4.77	1992	
rs*	-3.13	1979	ns	-4.21	1990	

2. Equation Estimated:  $X_t = a_1 + a_2X_{t-1} + a_3\Delta X_{t-1} + bt + c_1D_{1t} + c_4D_{4t} + u_t$

Where the dummy variable  $D_1$  has already been defined above and the dummy variable,  $D_4$  is defined as:  $D_4 = t - T_B$  for  $t > T_B$  and 0 otherwise, where  $T_B$  is the year of break.

\* significant at 5% level

\*\* significant at the 1% level

+ significant at the 10% level

**Table 2:** Results of Cointegration Tests by Johansen Method<sup>1</sup>

2A: Regressions without Dummy Variables

<b>Regression</b>	LR Trace Test Statistic for the null hypotheses of the number of cointegrating vectors			
<b>Regression</b>	Null Hypothesis <sup>1</sup> : $H_0 : nc = 0$ $H_0 : nc \leq 1$			
	Alternative Hypothesis: $H_1 : nc > 1$ $H_1 : nc \geq 2$			
	critical values at 5%(*) and 10%(+) level			
B, S, rs	48.40*	21.49	39.33	23.83
B, S, rl	51.07*	23.08	same as above	
B, NS, rs	48.60*	21.76	same as above	
B, NS, rl	51.30*	23.31	same as above	
b, ns, rs, g <sup>2</sup>	57.95 <sup>+</sup>	34.91	55.01	36.28
b, ns, g, rl <sup>2</sup>	64.41*	37.39	same as above	

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### 2B: CI Regressions without Dummy Variables

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<b>Regression</b>	LR Trace Test Statistic TB Dummy Variables			
	Null Hypothesis: $H_0 : nc = 0$ $H_0 : nc \leq 1$			
	Alternatives: $H_1 : nc \geq 1$ $H_1 : nc \geq 2$			
B, S, rs, D <sub>0</sub> , D <sub>1</sub> <sup>2</sup>	69.99	45.71	1989	D <sub>0</sub> = 1 for t > TB, 0 otherwise
				D <sub>1</sub> = S for t > TB, 0 otherwise
B, NS, rs, D <sub>0</sub> , D <sub>2</sub> <sup>2</sup>	76.12 <sup>+</sup>		41.07	1992 D <sub>2</sub> = NS for t > TB, 0 otherwise
				D <sub>0</sub> = 1 for t > TB, 0 otherwise
b, ns, rs, g, D <sub>0</sub> <sup>2</sup>	81.43 <sup>+</sup>	49.13	1989	
b, ns, rl, g, D <sub>0</sub>	86.02*	51.29		
10% critical values for above 4 equations for respective alternative hypotheses 77.55 55.01				
b, ns, rs, g, D <sub>0</sub> , D <sub>3</sub> <sup>3</sup>	53.41		31.15	1989 D <sub>3</sub> = g for t > TB, 0 otherwise
b, ns, rl, g, D <sub>0</sub> , D <sub>3</sub> <sup>3</sup>	98.85		44.48	1989
10% critical values for the above 2 equations 104.27 77.55				

1. To save space, we report the LRs for only the first two of the null hypotheses on number of CI vectors.
2. These have a unique CI vector at the 5% level by the LR test for maximum Eigen value



3. *These have a unique CI vector at the 10% level by the LR test for maximum Eigen value*  
\* *significant at 5% level*  
+ *significant at 10% level*

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