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Performance of a Random Number of Complex Systems in the Environment of a Random Number of Competing and Catastrophic Information Risks

By

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Abstract

The paper makes use of the concept of a random sum of nonnegative random variables and the concept of a minimum of a random number of nonnegative random variables in order to formulate a stochastic model. Sufficient conditions for embedding the formulated stochastic model into an important class of stochastic models are also established. Moreover, the paper establishes applications of the model in investigating the performance of a random number of complex systems being in the environment of a random number of competing and catastrophic information risks.

Keywords: Performance, Risk, System, Information

JEL Classification: C51

1. Introduction

The concept of a random sum of nonnegative, independent and identically distributed random variables is generally recognized as a very strong analytical tool of probability theory with particularly important applications in many significant practical disciplines (Ross, 1970). Moreover, the concept of a minimum of a random number of nonnegative, independent and identically distributed random variables, and the concept of a maximum of a random number of nonnegative independent and identically distributed random variables are also generally recognized as extremely useful for the theoretical foundation and practical applicability in various disciplines of extreme value theory (Beirlant et al. 1996 ; Embrechts et al. 1997 ; Embrechts et al. 1999). Recently, these three concepts have been used by Artikis and Artikis (2005), Artikis and Artikis (2007) and Artikis et al. (2007) for the formulation of stochastic discounting models.

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More recently, the concept of a minimum of a random number of nonnegative, independent and identically distributed random variables has been used by Artikis and Artikis (2008) in the formulation of a stochastic multiplicative model for investigating the performance of a complex system in the environment of a random number of competing and catastrophic risks. The present paper is mainly devoted to the extension of the theoretical and practical applicability of the stochastic multiplicative model formulated by Artikis and Artikis (2008).

More precisely, the paper concentrates on the implementation of three purposes. The first purpose is the formulation of a stochastic multiplicative model by making use of a random sum of nonnegative random variables and a minimum of a random number of nonnegative random variables. The second purpose is the establishment of sufficient conditions for embedding the characteristic function of the formulated stochastic multiplicative model into the class of characteristic functions corresponding to stochastic models based on the product of two nonnegative and independent random variables. The third purpose is the establishment of applications of the formulated stochastic multiplicative model in investigating the performance of a random number of complex systems being in the environment of a random number of competing and catastrophic risks.

The paper makes quite clear that the use of a fundamental concept of probability theory and the use of a fundamental concept of extreme value theory can substantially contribute to the investigation of the behaviour of complex systems. In particular, it is shown that these fundamental concepts can facilitate the development, evaluation and implementation of effective risk control and risk financing operations for such systems. This means that the formulated stochastic multiplicative model, based on a random sum of nonnegative random variables and a minimum of a random number of nonnegative random variables, can be a strong analytical tool for constructing risk management programs of complex systems. The basic analytical components and the mathematical structure of the formulated stochastic model provide risk managers with the ability to obtain effective combinations of risk control and risk financing operations suitable for treatment of risks threatening complex systems. A very interesting research area of modern risk management is the investigation of the evolution of complex systems under competing and catastrophic risks. The stochastic multiplicative model formulated by the present paper seems to be suitable for investigating the performance of a random number of complex systems being in the environment of a random number of competing and catastrophic risks. The practical applicability of the formulated stochastic multiplicative model becomes particularly significant when such a stochastic model is used for the description and analysis of complex systems under information risks. The strong dependence of complex systems on information systems makes necessary the investigation of the evolution of complex systems under conditions of information risks. It seems to be of some particular importance to consider the evolution of complex systems in the environment of competing and catastrophic information risks. Such a consideration of the evolution of complex systems provides new research directions in the discipline of modern risk management. Directions of this kind require the discipline of risk management to make use of results and methods from other practical disciplines. It is generally accepted that the use of results and methods from operational research, strategic management, informatics, reliability theory, statistics, probability theory, insurance and extreme value theory to risk management will extend the importance of risk identification, risk measurement and risk treatment operations. A significant part of the third purpose of the present paper is the establishment of applications of the formulated stochastic multiplicative model in investigating the performance of a random number of complex systems being in the environment of a random number of competing and catastrophic information risks. Such applications constitute very good theoretical and practical reasons for undertaking further research activities in the area of formulating and investigating stochastic models describing fundamental operations of the risk management process.

In this paper the term complex system means a set of intertangled networks of human brains engaged in the process of solving problems within such a set (Kervern, 1994).

2. Formulating the Stochastic Model

The present section makes use of the concept of a random sum of nonnegative random variables and the concept of a minimum of a random number of nonnegative random variables in order to formulate a stochastic multiplicative model.

We suppose that

$$R$$

is a discrete random variable with values in the set

$$\mathbf{N}_0 = \{0,1,2,...\}$$

and

$$\{S_r, r = 1,2,...\}$$

is a sequence of nonnegative random variables. We consider the random sum

$$L = S_1 + S_2 + ... + S_R .$$

We also suppose that

$$N$$

is a discrete random variable with values in the set

$$\mathbf{N} = \{1,2,...\}$$

and

$$\{X_n, n = 1,2,...\}$$

is a sequence of nonnegative random variables. We consider the random variable

$$T = \min\{X_1, X_2, ..., X_N\}.$$

The present paper is mainly devoted to the applicability of the stochastic multiplicative model

$$Y = LT$$

in investigating fundamental operations of complex systems arising in various practical disciplines. The applicability of the above stochastic model is substantially extended by establishing sufficient conditions for the derivation of the corresponding distribution function.

3. Characteristic Function of the Stochastic Model

The present section concentrates on the investigation of the distribution function of the formulated stochastic model. Since the evaluation of such a distribution function is impossible it is advisable to evaluate the corresponding characteristic function.

We suppose that the discrete random variable

$$R$$

has probability generating function

$$P_R(z)$$

and the random variables of the sequence

$$\{S_r, r = 1,2,...\}$$

are independent and distributed as the random variable

$$S$$

with characteristic function

$$\varphi_S(u).$$

We also suppose that the discrete random variable

$$N$$

has probability generating function

$$P_N(z)$$

and the random variables of the sequence

$$\{X_n, n = 1,2,...\}$$

are independent and distributed as the random variable

$$X$$

with distribution function

$$F_X(x).$$

If

$$R\,,\ \{S_r, r = 1,2,...\},\ N\,,\ \{X_n, n = 1,2,...\}$$

are independent then from Artikis and Artikis (2007) it follows that the random variables

$$L = S_1 + S_2 + ... + S_R\,,\ T = \min\{X_1, X_2,..., X_N\}$$

are independent. Hence, it is readily shown that the characteristic function of the stochastic model

$$Y = LT$$

is given by

$$\varphi_Y(u) = \int_0^{\infty} P_R(\varphi_S(ut))\ d\{1 - P_N(1 - F_X(t))\}.$$

The use of the above characteristic function provides a method for evaluating the distribution function of the formulated stochastic multiplicative model. The method is based on the Fourier inversion of the characteristic function of Y. More precisely, the modeller can obtain the distribution function of Y by first obtaining and then inverting the characteristic function corresponding to Y. In addition, the use of a computational algorithm known as the fast Fourier transform makes the inversion of the above characteristic function more manageable by greatly reducing computational time. It is generally accepted that Fourier analysis is a very powerful methodology and it could eventually facilitate the investigation of distribution functions corresponding to complicated stochastic models. Application of Fourier analysis in stochastic modelling has been slowed by the relatively advanced nature of mathematics it entails and by the absence of computer programs that have been adapted for convenient use in various practical disciplines. As these barriers are overcome, application of Fourier analysis to solve problems in stochastic modelling will become more common.

4. Applications of the Stochastic Model in Information Risks

The present section concentrates on the establishment of applications of the formulated stochastic model in the area of information risk evaluation operations.

We assume that the discrete random variable

$$N$$

denotes the number of catastrophic risks threatening a random number

$$R$$

of complex systems at a future time point being the beginning of their functioning. The assumptions that N and R are random variables are supported by the assumption that the incorporated systems are complex. We assume that the random variable

$$X_n, n = 1,2,...$$

denotes the occurrence time of the nth catastrophic risk. The random variable

$$T = \min\{X_1, X_2, ..., X_N\}$$

denotes the smallest catastrophic risk occurrence time. Moreover, we assume that the random variable

$$S_r, r = 1,2,...$$

denotes the performance of the rth complex system per unit of time. Hence the random sum

$$L = S_1 + S_2 + ... + S_R$$

denotes the performance of R such complex systems per unit of time. The random variable

$$T = \min\{X_1, X_2, ..., X_N\}$$

gives analysts and modellers the ability to consider these complex systems in the environment of N competing and catastrophic risks. It is readily understood that the stochastic multiplicative model

$$Y = LT$$

denotes the performance of these R complex systems during their lifetime

$$T = \min\{X_1, X_2, ..., X_N\}.$$

The mathematical structure of the formulated stochastic model and the sufficient conditions for evaluating the corresponding characteristic function provide risk managers with strong analytical tools for investigating the performance of a random number of complex systems being in the environment of a random number of independent, competing and catastrophic risks. More precisely, the presence of the random sum of nonnegative random variables

$$L = S_1 + S_2 + ... + S_R$$

and the presence of the minimum of a random number of nonnegative random variables

$$T = \min\{X_1, X_2, ..., X_N\}$$

in the mathematical structure of the stochastic multiplicative model

$$Y = LT$$

can substantially facilitate the process for making proactive risk treatment decisions. It is generally recognized that decisions of this kind constitute extremely important parts of any proactive risk management program. The contribution of proactive risk management programs to the evolution of complex systems is recognized by the international risk management community as being of great practical importance. From the fact that the mathematical structure and the properties of the formulated stochastic model can support proactive risk management decision making it follows that the theoretical and practical results of the present paper are very useful for developing, assessing and implementing proactive risk management programs. It is easily demonstrated that the incorporation of a fundamental concept of probability theory and a fundamental concept of extreme value theory in the formulation of the above stochastic multiplicative model provide risk analysts, risk modellers and risk managers with an analytical tool which seems to be very important for investigating the performance of a random number of complex systems being in the environment of a random number of independent, competing and catastrophic risks.

From the fact that information systems are extremely useful for complex systems, it follows that it is particularly important to consider the application of the formulated stochastic multiplicative model in investigating the performance of a random number of complex systems being in the environment of a random number of competing and catastrophic information risks. The term information system means a system of persons, data records and activities that process the data and information in a given complex system, including manual processes or automated processes. Moreover, the term information risk means a risk of which the cause can damage an information system. Information risk management may be defined as the systematic process of managing the information risks of a complex system to achieve its objectives in a manner consistent with public interest, human safety, environmental factors, and the law. It consists of the planning, organizing, directing, and controlling activities undertaken with the intent of providing an efficient proactive information risk plan that reduces the adverse impact of information risks on a complex system.

During the last three decades risk modelers, risk analysts and risk managers have responded actively and creatively to information risks. The strong dependence of complex systems on information systems makes necessary a comment on the great importance of stochastic modeling activities for describing, analyzing and implementing information risk management operations. Information risk causes such a fire, flood, earthquake, electrical and magnetic disturbances, change in temperature and humidity, hardware and software failure, human error, and criminal action continuously threaten the proper function of an information system. Undoubtedly, these information risk causes can generate an information system breakdown which subsequently can have a serious impact on the continuity of the activities of the corresponding complex system. It is readily understood that when it comes to protecting information systems from the adverse effects of these information risk causes, risk managers should emphasize information control and information risk financing operations. For information systems, information risk control operations constitute the number one priority, and risk financing operations come second. Catastrophic information risks require the use of information risk financing operations. Empirical studies, concerning the risk management practices for complex systems have shown that risk managers have begun to treat information risks by making information risk control operations very important parts of proactive information risk management programs (Meulbroek, 2002). From the results of these studies, we can easily get the conclusion that stochastic modelling activities can be of great importance in assessing, selecting and implementing information risk management operations.

An interpretation in information risk management of the formulated stochastic multiplicative model is the following. We assume that the discrete random variable

$$N$$

denotes the number of catastrophic risks threatening an information system which supports a random number

$$R$$

of complex systems at a future time point being the beginning of their functioning. We assume that the random variable

$$X_n, n = 1,2,...$$

denotes the occurrence time of the nth catastrophic information risk. The random variable

$$T = \min\{X_1, X_2,..., X_N\}$$

denotes the smallest catastrophic information risk occurrence time. Moreover, we assume that the random variable

$$S_r, r = 1,2,...$$

denotes the performance of the rth complex system per unit of time. Hence the random sum

$$L = S_1 + S_2 + ... + S_R$$

denotes the performance of R such complex systems per unit of time. Since the smallest catastrophic information risk occurrence time

$$T = \min\{X_1, X_2,..., X_N\}$$

is also the lifetime of these R complex systems, then the stochastic multiplicative model

$$Y = LT$$

denotes the performance of such systems during their lifetime. It is readily understood that the above stochastic model can contribute to the improvement of the information risk treatment operations.

5. Conclusions

By incorporating the concept of a random sum of nonnegative random variables and a minimum of a random number of nonnegative random variables we formulated a stochastic multiplicative model. It is shown that the mathematical structure of the formulated stochastic model is suitable for investigating the performance of a random number of complex systems under a random number of competing and catastrophic information risks.

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