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Bernoulli Selecting Processes in Actuarial Decisions

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Abstract

Bernoulli selecting processes are generally considered as valuable analytical tools for making decisions in many disciplines of particular theoretical and practical importance. The present paper concentrates on the formulation, investigation and actuarial applications of a stochastic model describing a Bernoulli selecting process. It is shown that the formulated stochastic model can substantially support the applicability of such a selecting process for making insurance decisions incorporating significant elements of proactivity.

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JEL Classification: *C51*

1. Introduction

Stochastic discounting models and Bernoulli selecting processes constitute very strong analytical tools for describing, analyzing and solving significant problems arising in economics, management, insurance, operational research and other important practical disciplines (Rolski et al., 2000). The formulation of stochastic models based on Bernoulli selecting processes and the formulation of Bernoulli selecting processes based on stochastic discounting models are two recent and interesting research areas (Artikis et al., 1997; Jerwood and Moshakis, 1997; Artikis, 2010). Research activities in these areas can provide designers, modellers, and analysts of modern complex organizations with probabilistic information particularly useful for making optimal decisions. The present paper formulates a modification of a stochastic model describing a Bernoulli selecting process (Artikis et al., 1997). Moreover, the paper establishes properties and actuarial applications of the formulated stochastic model describing a Bernoulli selecting process and based on a stochastic discounting model.

The following three purposes are implemented by the present paper. The first purpose is the formulation of a stochastic model describing a Bernoulli selecting process by making use of a discrete random variable, a sequence of nonnegative and identically distributed random variables, two nonnegative random variables, and a Bernoulli random variable. The second purpose is the establishment of sufficient conditions for embedding the characteristic function of the formulated stochastic model into a significant class of finite mixtures of characteristic functions. The third purpose is the establishment applications of the formulated stochastic model in making decisions for undertaking risks by an insurance company. It can be

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said that the implementation of the three purposes of the present paper facilitates the introduction of proactivity concepts to the discipline of insurance (Kloman, 1992; Meulbroek, 2002; Vaugham, 1996).

2. Formulation of a Stochastic Model

The present section of the paper makes use of a discrete random variable, a sequence of nonnegative and identically distributed random variables, two nonnegative random variables and a Bernoulli random variable in order to formulate a stochastic model. This section provides a modification of a known stochastic model (Artikis et al., 1997).

We consider the discrete random variable *N* with values in the set $\mathbf{N}_0 = \{0,1,2,...\}$ and probability generating function $P_N(z)$. We also consider the sequence of nonnegative and independent random variables $\{X_n, n=1,2,...\}$ distributed as the random variable X with characteristic function $\varphi_X(u)$. We introduce the random sum $V = X_1 + X_2 + ... + X_N$.

Moreover, we consider the nonnegative random variable *T* with distribution function $F_T(t)$ and the nonnegative real number *r*. We introduce the random variable $S = Ve^{-rT}$ .We also consider the nonnegative random variable *Y* with characteristic function $\varphi_Y(u)$ and the Bernoulli random variable *C* with probability generating function $P_C(z) = pz + q$, $0 < p < 1$, $q = 1 - p$.

We introduce the stochastic model

$$L = \begin{cases} Y & , \quad C = 0 \\ & \\ S & , \quad C = 1 \end{cases} .$$

An interpretation of the formulated stochastic model, in the area of Bernoulli selecting processes, is the following. We suppose that the random variable *N* denotes the number of cash flows arising at the random time point *T* and the random variable $X_n, n = 1,2,...$ denotes the size of the *n* th cash flow. It is easily shown that the random sum $V = X_1 + X_2 + ... + X_N$ denotes the total size of the N cash flows arising at the random time point *T* with probability p .If the nonnegative real number *r* denotes force of interest then the random variable $S = Ve^{-rT}$ denotes the present value, as viewed from the time point 0, of the total size $V = X_1 + X_2 + ... + X_N$ of the *N* cash flows arising at the random time point *T* with probability p .Moreover, we suppose that the random variable *Y* denotes the size of a cash flow arising at the time point 0 with probability q . It is obvious that the stochastic model:

$$L = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases}$$

denotes the size of the cash flow corresponding to the time point 0 . From the fact that the mathematical structure of the stochastic model

$$L = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases}$$

incorporates the Bernoulli random variable C and the random sum $V = X_1 + X_2 + \ldots + X_N$, it follows that the establishment of sufficient conditions for evaluating the corresponding characteristic function $\varphi_V\left(u\right)$ can substantially support the applicability of this model in various practical disciplines.

3. Characteristic Function of a Stochastic Model

The establishment of sufficient conditions for evaluating the characteristic function of the formulated stochastic model is the purpose of the present section of the paper. It is advisable to evaluate this characteristic function since an evaluation of the corresponding distribution function is not possible.

If N $\left\{X_n, n = 1,2,\ldots\right\}$ *T, Y, C,* are independent, then it follows that the random variables *Y, C,* are independent, N $\left\{X_n, n = 1,2,\ldots\right\}$ are independent, the random variables $V = X_1 + X_2 + \ldots + X_N$, *T, C* are independent and the random variables $V = X_1 + X_2 + \ldots + X_N$, $W = e^{-rT}$, C , are also independent. Hence, the random variables $S = Ve^{-rT}$, *C,* are independent and it can be easily shown that

$$\varphi_L\left(u\right) = q\varphi_Y\left(u\right) + p\int_0^1 P_N\left(\varphi_X\left(uw\right)\right) d\left[1 - F_T\left(-\frac{1}{r}\log w\right)\right]$$

is the characteristic function of the stochastic model

$$L = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases}.$$

It is easily understood that the theoretical results of the present section of the paper substantially facilitate the applicability in various practical disciplines of the formulated stochastic model.

4. Actuarial Applications of a Stochastic Model

The interpretation of the formulated stochastic model as a valuable analytical tool of the actuarial discipline is the purpose of the present section of the paper.

We suppose that the discrete random variable *N* denotes a number of risks at the random time point *T* and the nonnegative random variable $X_n, n = 1, 2, ...$ denotes the severity of the *n* the risk. It is easily seen that the random sum $V = X_1 + X_2 + ... + X_N$ denotes the severity of the *N* risks at the random time point T .Hence, the random variable $S = Ve^{-rT}$ denotes the present value of such a severity, as viewed from the time point 0. We also suppose that the nonnegative random variable *Y* denotes the severity of another risk at the time point 0 .

Moreover, we suppose that the *N* risks with severity $V = X_1 + X_2 + ... + X_N$ can be undertaken with probability *p* by an insurance company at the random time point *T* or the risk with severity *Y* can be undertaken with probability *q* by the same insurance company at the time point 0. Hence, the stochastic model

$$L = \begin{cases} Y & , \quad C = 0 \\ \\ S & , \quad C = 1 \end{cases}$$

denotes the obligation undertaken by the insurance company at the time point 0. It is quite obvious that the above interpretation, in the actuarial discipline, of the formulated stochastic model can provide various risk experts with very significant probabilistic information for making insurance decisions.

5. Conclusions

The theoretical contribution of the present paper consists of the formulation and investigation of a stochastic model describing a Bernoulli selecting process. In addition, the practical contribution of the paper consists of providing applications of the formulated stochastic model in actuarial decision making. It can be said that the theoretical and practical results of the paper facilitate the applicability of proactive operations in the discipline of insurance.

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