Sustainable Growth, the Budget Deficit, and Inflation

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Abstract

This paper analyses the dynamics of economic growth when the government deficit is money financed. Our model specifies capital formation as resulting from internally generated non-traded investment expenditure. The central innovation of this paper is the specification of the long run equilibrium conditions as those of zero acceleration of inflation and capital formation. This gives rise to positive inflation and sustainable growth (positive capital formation). We found there exists multiple economic growth equilibria. One of these is associated with high growth and low inflation; the other with high inflation and low growth. Furthermore, there is policy hysteresis, which has important implications for the current debates on fiscal policy.

Key words: growth, investment, inflation, budget deficits.

J. E. Lit. Nos.: E22, E31, H60, H62, O40

1. Introduction

This paper analyses the dynamics of economic growth when the government deficit is money financed. We follow the approach of Brock (1996), Obstfeld and Stockman (1985), Dornbusch (1980) and Fischer and Frenkel (1972) by specifying capital formation in terms of internally generated non-traded investment expenditure, which represents growth in our model. A key feature of this paper, distinguishing it from the existing growth literature, is positive inflation and growth (capital formation) in the long run. In the traditional literature, such

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as the dynamic versions of Ramsey (1928) and Solow (1956) surveyed in Blanchard and Fischer (1989), the long run equilibrium conditions are zero inflation and zero growth. In our model, the long run equilibrium conditions are zero acceleration of inflation and capital formation.

Using the framework specified above, we attempt to analyse the effects of pure monetarisation of the budget deficit, as represented by the Cagan (1956) money demand function. Moreover, our analysis extends and complements recent work on money financing of budget deficits by Shah (1984), George and Oxley (1991) following the seminal analysis of Blinder and Solow (1973) [1]. We show that continued monetarisation of the budget deficit gives a possibility of two stable equilibria depending on the magnitude of the monetarisation. Modest monetarisation may give rise to a low inflation, high growth equilibrium. On the other hand, more aggressive monetarisation may lead to a high inflation, low growth equilibrium. This possibility is a policy lesson to countries contemplating high levels of money finance of their budget deficit. This is a relevant concern, particularly for developing countries, in pursuit of elusive high growth. Thus, the paper warns of the consequences of policy hysteresis in the context of the ongoing debate on the monetarisation of the budget deficit in developing countries.

In section two we outline the model. The final section presents our conclusions and suggests directions for future research.

2. Money Financed Government Deficits

We specify the real government budget constraint as

$$G = \dot{M} \tag{1}$$

where M is the real money supply as a proportion of real income, $G = G(k,\pi,F)$ is the real government deficit as a proportion of real income, which is a function of k, the growth rate[2], π, the inflation rate[3], and F, a shock parameter reflecting the impact of some fiscal shock[4] on the budget deficit. The government deficit is comprised of expenditure, G^E, and revenues, G^R, according to:

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[1] Blinder and Solow (1973) also consider bond financing the government deficit.

[2] The growth rate is defined as $k \equiv \dot{K}/K$ where K is the physical capital stock, which is pursued further by Gazioglu and McCausland (1997).

[3] The inflation rate is defined as $\pi \equiv \dot{P}/P$ where P is the price level.

[4] By 'fiscal shock' we mean an exogenous shock that requires an increase in fiscal expenditure.

$$G = G(k,\pi,F) = G^E(\pi,F) - G^R(k,\pi) \tag{2}$$

Rises in real growth improve revenues, $G_k^R > 0$, reducing the real government deficit, $G_k < 0$. The fiscal shock by definition increases government expenditure, $G_F^E > 0$, hence increasing the government deficit, $G_F > 0$. Inflation both raises expenditure, $G_\pi^E > 0$, and raises revenue, $G_\pi^R > 0$, thus the effect of inflation on the government debt may take either sign depending on whether the revenue or expenditure effects dominate, thus $G_\pi < 0$ or $G_\pi > 0$. It is this sign switch that gives rise to the non-linearities which generate the multiple equilibria that we discuss later.

We use the Cagan money demand function, which states that money demand as a proportion of real income is a negative function of inflation, which in general form may be expressed as

$$M = M(\pi) \Rightarrow \dot{M} = M'(\pi)\dot{\pi} \Rightarrow \dot{M} = \dot{M}(\pi,\dot{\pi}) \tag{3}$$

where $M_\pi < 0$ and hence[5] $\dot{M}_\pi > 0$ and $\dot{M}_{\dot{\pi}} < 0$. Thus, combining equations (1), (2) and (3) gives

$$\dot{M}\left(\overset{+}{\pi},\overset{-}{\dot{\pi}}\right) = G(\overset{-}{k},\overset{+/-}{\pi},\overset{+}{F}) \tag{4}$$

Solving for $\dot{\pi}$ we obtain

$$\dot{\pi} = H(\overset{+}{k},\overset{-/+}{\pi},\overset{-}{F}) \tag{5}$$

where $H_k > 0$, $H_F < 0$ and either $H_\pi < 0$ or $H_\pi > 0$. Hence $\dot{\pi}_k > 0$, $\dot{\pi}_F < 0$ and either $\dot{\pi}_\pi < 0$ or $\dot{\pi}_\pi > 0$. If expenditure effects, G_π^E, dominate revenue effects, G_π^R, then $G_\pi > 0$ and hence $\dot{\pi}_\pi < 0$. One explanation for this is that increased expenditures on social security payments following the onset of inflation take place immediately whereas increased tax receipts feed through later but at an inflation-eroded value.

We specify real domestic trading profits, *Q*, as being of the form

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5 The conventional form of the Cagan money demand function is $M = A\exp(-\alpha\pi)$. Therefore, $\dot{M} = -\alpha A\dot{\pi}\exp(-\alpha\pi)$, which implies that $\dot{M}_\pi > 0$ and $\dot{M}_{\dot{\pi}} < 0$.

$$Q = Q(\overset{+}{k}, \overset{+}{\pi}, \overset{+}{F}) \tag{6}$$

where $0 < Q_k < 1$ since growth (of the capital stock) raises revenues, $Q_\pi > 0$ since unexpected inflation improves profits, and $Q_U > 0$ since fiscal expenditure raises revenues. Unexpected inflation improves profits since it involves a redistribution from lenders to borrowers (where the firms are assumed to be the borrowers), thus the growth of real profits, $q \equiv \dot{Q}/Q$, exceeds inflation, $\pi \equiv \dot{P}/P$.

A proportion, $\gamma < 1$, of these profits are used to augment the domestic capital stock, K, so we may write growth, $k \equiv \dot{K}/K$ as

$$k = \gamma Q(\overset{+}{k}, \overset{+}{\pi}, \overset{+}{F}) \tag{7}$$

Hence[6]

$$\dot{k} = \phi(\overset{+}{k}, \overset{+}{\pi}, \overset{+}{F}) \tag{8}$$

Equations (5) and (8) thus completely describe the system which may be represented as

$$\begin{bmatrix} \dot{k} \\ \dot{\pi} \end{bmatrix} = \begin{bmatrix} \dot{k}_k & \dot{k}_\pi \\ \dot{\pi}_k & \dot{\pi}_\pi \end{bmatrix} \begin{bmatrix} k \\ \pi \end{bmatrix} + \begin{bmatrix} \dot{k}_F \\ \dot{\pi}_F \end{bmatrix} [F] \tag{9}$$

This is illustrated using the phase diagram presented in Figure 1 below[7]. Each equilibrium[8] corresponds to a situation of both non-accelerating growth

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6 This follows because $\dot{k} = \gamma[Q_k(k)\dot{k} + Q_\pi(\pi)\dot{\pi} + Q_F(F)\dot{F}]$, from equation (7), which upon rearrangement yields $\dot{k} = \gamma\left[\overset{+}{Q_\pi}(\pi)\dot{\pi} + \overset{+}{Q_F}(F)\dot{F}\right] \Big/ \left[1 - \gamma \overset{+}{Q_k}(k)\right]$, which gives, noting that $\gamma Q_k < 1$, the functional form $\dot{k} = \phi(\overset{+}{k}, \overset{+}{\pi}, \overset{+}{F}, \dot{\pi}, \dot{F})$, which, with $\dot{\pi} = \dot{F} = 0$, gives the form shown in equation (8).

7 The arrows of motion are determined in the conventional fashion locally around each equilibrium. Full details are to be found in the Appendix.

8 Table summarising the properties of the three equilibria:

E^L (stable)	• $G_\pi^R > 0$	and $G_\pi^E > G_\pi^R$	$G_\pi > 0$	$\dot{\pi}_\pi < 0$ (stable)
E^M (unstable)	$G_\pi^R > 0$	and $G_\pi^E < G_\pi^R$	$G_\pi < 0$	$\dot{\pi}_\pi > 0$ (unstable)
E^H (stable)	$G_\pi^R < 0$	implies directly that	$G_\pi > 0$	$\dot{\pi}_\pi < 0$ (stable)

and non-accelerating inflation. When inflation is relatively low (equilibrium E^L in Figure 1) it is relatively easy for governments to increase expenditure in line with inflation (the term G_π^E in equation (2) is relatively strong) whilst at the same time, although revenues increase as people move into higher tax brackets due to the inflation (captured by the term G_π^R in equation (2)), this effect is relatively weak and lagged. Thus from equation (2) $G_\pi^E > G_\pi^R$, and hence $G_\pi > 0$. This implies, from equations (4) and (5), that $\dot{\pi}_\pi < 0$ which gives a decelerating inflation and hence stable solution.

When inflation rises to moderate levels (equilibrium E^M in Figure 1), governments find it difficult to keep pace with inflation in terms of increasing expenditure and tax thresholds, thus G_π^E in equation (2) is relatively weak (expenditure such as social security benefits not keeping pace with inflation) and G_π^R in equation (2) is relatively strong (people entering higher tax brackets). Thus, from equation (2), $G_\pi^E < G_\pi^R$ and hence $G_\pi < 0$. This implies, from equations (4) and (5), that $\dot{\pi}_\pi > 0$ which gives an accelerating inflation and hence an unstable solution.

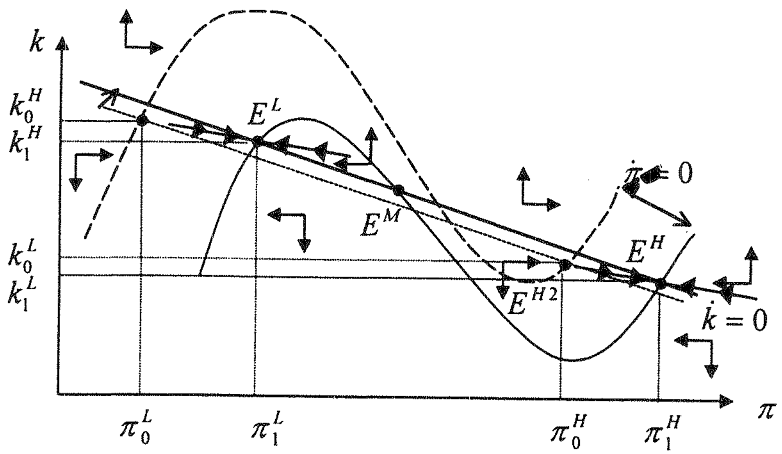

Figure 1

When inflation rises to very high levels (equilibrium E^H in Figure 1), the inflation tends to reduce tax revenues, implying $G_R < 0$ in equation (2) and hence $G_\pi > 0$. Again this implies, from equations (4) and (5), that $\dot{\pi}_\pi < 0$, which gives a decelerating inflation and hence a second stable solution.

The stable equilibrium E^L seems to correspond to a situation with a desirable mix of high growth and low inflation. However, the existence of a second stable equilibrium E^H, corresponding to a position of high inflation and low growth of capital stock, suggests the danger of policies of excessive monetarisation of the budget deficit. In Figure 1 we illustrate the effect of a 'small' rise in money financed government expenditure, F. This shifts the $\dot{k}=0$ locus upwards and the $\dot{\pi}=0$ locus downwards (as shown in equations (16) and (17) in the Appendix) resulting in a long run rise in inflation (as shown in equation (14) in the Appendix), and a fall in the growth of capital stock (providing $\pi_F > k_F$ from equation (15) in the Appendix, that is, the inflation rate exceeds the growth rate in response to the fiscal shock, which is likely since this expenditure is money financed).

In Figure 2 we illustrate the effect of a 'large' fiscal shock. We can see that this results in the loss of the low inflation-high growth equilibrium, E^L, and a switch to the high inflation-low growth equilibrium, E^H. We have made the standard qualitative distinction between 'small' and 'large' changes in terms of 'large' changes being those exceeding the critical value which results in one of the stable equilibria being lost. This implies that there is policy irreversibility.

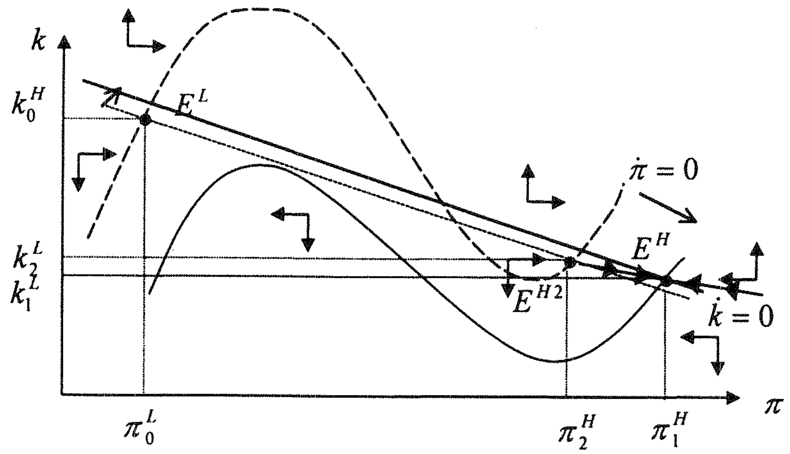

Figure 2

Hysteresis Case

A cut in money financed expenditure, F, to its former level will *not* result the restoration of the initial equilibrium, E^L, but will result in a move to the locally proximate equilibrium E^{H2}, which also has the characteristics of high inflation and low growth. To restore the initial equilibrium, E^L, we would need a much larger cut in money financed expenditure, F, which is usually politically difficult and socially damaging.

Concluding Comments

This paper constructs a model in which the government deficit is money financed, and in which capital formation is internally generated from non-traded investment expenditure. The central innovation of this paper is the specification of the long run equilibrium conditions as those of zero *acceleration* of inflation and capital formation. This gives rise to positive inflation and sustainable growth (positive capital formation). In terms of policy, we consider the monetarisation of the government deficit following a fiscal shock. We show that there is a possibility of two stable long run equilibria. The first corresponds to high inflation and low growth (of capital stock) and the other corresponds to low inflation and high growth. We show that monetarisation leads to higher inflation and potentially lower growth, and that excessive monetarisation leads to a switch from a low inflation-high growth equilibrium to a high inflation-low growth equilibrium. Furthermore, there is policy irreversibility, in the sense that the low inflation-high growth equilibrium can only be restored by even larger cuts in the deficit, which though desirable, may be politically constrained, and hence a smaller, but insufficient cut, may lead the country to become stuck in a high inflation-low growth trap.

Appendix

The slopes of the stationary loci $\dot{\pi} = 0$ and $\dot{k} = 0$ are, using the implicit function rule, given by the following equations

$$\left.\frac{\partial k}{\partial \pi}\right|_{\dot{\pi}=0} = \frac{-\overset{-S}{\dot{\pi}_{\pi}}}{\underset{+}{\dot{\pi}_{k}}} > 0^{S} \qquad (10)$$

$$\left.\frac{\partial k}{\partial \pi}\right|_{\dot{k}=0} = \frac{-\overset{+}{\dot{k}_{\pi}}}{\underset{+}{\dot{k}_{k}}} < 0 \qquad (11)$$

The conditions that the equilibria are stable saddle-path equilibria are firstly that the determinant of the system is negative. Secondly, the trace of the system must be negative.

$$\det \mathbf{J} = \underset{+}{\dot{k}_k}\,\underset{-S}{\dot{\pi}_\pi} - \underset{+}{\dot{k}_\pi}\,\underset{+}{\dot{\pi}_k} < 0^S \tag{12}$$

$$\mathrm{tr}\mathbf{J} = \underset{+}{\dot{k}_k} + \underset{-S}{\dot{\pi}_\pi} < 0^{S^*} \tag{13}$$

The matrix **J** is that defined in equation (9). The condition in (12) is unambiguously met in the stable case. We assume that, for condition (13) to hold $|\dot{\pi}_\pi| > |\dot{k}_k|$.

The effect of the fiscal shock (changes in F) on equilibrium growth, $\tilde{k}$, and inflation, $\tilde{\pi}$, can be easily determined from (9) to be

$$\tilde{\pi}_F = \frac{|J_{\pi F}|}{|J|} = \frac{\overset{+}{\dot{k}_k}\,\overset{-}{\dot{\pi}_F} - \overset{+}{\dot{k}_F}\,\overset{+}{\dot{\pi}_k}}{\underset{+}{\dot{k}_k}\,\underset{-S}{\dot{\pi}_\pi} - \underset{+}{\dot{k}_\pi}\,\underset{+}{\dot{\pi}_k}} > 0^S \tag{14}$$

$$\tilde{k}_F = \frac{|J_{kF}|}{|J|} = \frac{\overset{+}{\dot{k}_F}\,\overset{-S}{\dot{\pi}_\pi} - \overset{+}{\dot{k}_\pi}\,\overset{-}{\dot{\pi}_F}}{\underset{+}{\dot{k}_k}\,\underset{-S}{\dot{\pi}_\pi} - \underset{+}{\dot{k}_\pi}\,\underset{+}{\dot{\pi}_k}} < 0^{S^*} \tag{15}$$

To obtain the sign reported in equation (15) we require $\dot{k}_F / \dot{k}_\pi < \dot{\pi}_F / \dot{\pi}_\pi$, that is $k_F > \pi_F$.

A rise in F shifts the $\dot{k} = 0$ upwards and the $\dot{\pi} = 0$ loci downwards.

$$\left.\frac{\partial k}{\partial F}\right|_{\dot{\pi}=0} = \frac{\overset{-}{\dot{\pi}_F}}{\underset{+}{\dot{\pi}_k}} < 0 \tag{16}$$

$$\left.\frac{\partial k}{\partial F}\right|_{\dot{k}=0} = \frac{\overset{+}{\dot{k}_F}}{\underset{+}{\dot{k}_k}} > 0 \tag{17}$$

Intuitively, for the $\dot{k}=0$ locus, for any given inflation rate, a rise in F generates a rise in growth. For the $\dot{\pi}=0$ locus, for any given inflation rate, a rise in F generates a fall in growth.

Finally, from equation (9)

$$\begin{bmatrix} \dot{k}_k & \dot{k}_\pi \\ \dot{\pi}_k & \dot{\pi}_\pi \end{bmatrix} \begin{bmatrix} \theta^S \\ 1 \end{bmatrix} = \rho^S \begin{bmatrix} \theta^S \\ 1 \end{bmatrix} \tag{18}$$

where θ^S is the stable eigen vector and ρ^S is the negative eigen value. Thus the slope of the stable manifold is given by θ^S which is less than the slope of the $\dot{k}=0$ locus.

$$\theta_S = \frac{-\dot{k}_\pi}{\dot{k}_k - \rho^S} < \text{slope}(\dot{k}=0) = \frac{-\dot{k}_\pi}{\dot{k}_k} < 0 \tag{19}$$

comprised of a revenue element, Q^R, and a cost element, Q^C, of the form

$$Q = Q^R(k,\pi,U) - Q^C(\pi) \tag{20}$$

[9]where $Q_k^R > 0$ since growth raises revenues, $Q_\pi^R > 0$ since inflation improves revenues, $Q_U^R > 0$ since reunification expenditure raises revenues, and $Q_\pi^C > 0$ since inflation increases costs. Hence we may write

$$Q = Q(\overset{+}{k}, \overset{+/-}{\pi}, \overset{+}{U}) \tag{21}$$

where $Q_k > 0$ since growth raises revenues, $Q_U > 0$ since reunification expenditure raises revenues, and either $Q_\pi > 0$ or $Q_\pi < 0$ depending on whether the revenue or cost effects dominate.

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[9] We assume an imperfectly competitive trading environment where, given constant stock market valuation of capital, profits are directly proportional to the real return on capital (the real interest rate), following Blanchard (1981).

Symbols List

A	constant
E	equilibrium (superscripts L, M, H denote low, middle, high inflation)
F	fiscal shock
G	real government deficit (superscripts E, R denote expenditure, revenue)
H	function defined in equation (5)
K	physical capital stock
M	real money supply as a proportion of real income
P	price level
Q	real domestic trading profits
J	matrix defined in equation (9)
S	stable solution (as superscript)
k	real growth rate $k \equiv \dot{K}/K$
q	real growth rate of domestic trading profits $q \equiv \dot{Q}/Q$
α (alpha)	constant
θ (theta)	stable eigen vector
ρ (rho)	negative eigen value
π (pi)	inflation $\pi \equiv \dot{P}/P$
∂ (delta 2)	partial differential operator
~	equilibrium (used above a symbol)
*	signing not ambiguous

Throughout the text, subscripts denote partial derivatives.

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